16–9. When only two gears are in mesh, the driving gear $A$ and the driven gear $B$ will always turn in opposite directions. In order to get them to turn in the same direction an idler gear $C$ is used. In the case shown, determine the angular velocity of gear $B$ when, if gear $A$ starts from rest and has an angular acceleration of $\alpha_A = (3t + 2) \text{ rad/s}^2$, where $t$ is in seconds.

$$\int_0^{\omega_b} d\omega_A = \int_0^t (3t + 2) \, dt$$

$$\omega_A = 1.5t^2 + 2t|_{t=5} = 47.5 \text{ rad/s}$$

$$(47.5)(50) = \omega_C (50)$$

$$\omega_C = 47.5 \text{ rad/s}$$

$$\omega_B (75) = 47.5(50)$$

$$\omega_B = 31.7 \text{ rad/s}$$

Ans.

16–10. During a gust of wind, the blades of the windmill are given an angular acceleration of $\alpha = (0.2\theta) \text{ rad/s}^2$, where $\theta$ is in radians. If initially the blades have an angular velocity of $5 \text{ rad/s}$, determine the speed of point $P$, located at the tip of one of the blades, just after the blade has turned two revolutions.

Angular Motion: The angular velocity of the blade can be obtained by applying Eq. 16–4.

$$\omega d\omega = \alpha d\theta$$

$$\int_0^\omega \omega d\omega = \int_0^{2\pi} 0.2\theta d\theta$$

$$\omega = 7.522 \text{ rad/s}$$

Motion of $P$: The speed of point $P$ can be determined using Eq. 16–8.

$$v_P = \omega r_P = 7.522(2.5) = 18.8 \text{ ft/s}$$

Ans.
16–22. The disk is originally rotating at \( \omega_0 = 8 \text{ rad/s} \). If it is subjected to a constant angular acceleration of \( \alpha = 6 \text{ rad/s}^2 \), determine the magnitudes of the velocity and the \( n \) and \( t \) components of acceleration of point \( B \) just after the wheel undergoes 2 revolutions.

\[
\omega^2 = \omega_0^2 + 2\alpha (\theta - \theta_0) \\
\omega^2 = (8)^2 + 2(6)(2\pi - 0) \\
\omega = 14.66 \text{ rad/s} \\
v_B = \omega r = 14.66(1.5) = 22.0 \text{ ft/s} \quad \text{Ans.} \\
(a_B)_t = \alpha r = 6(1.5) = 9.00 \text{ ft/s}^2 \quad \text{Ans.} \\
(a_B)_n = \omega^2 r = (14.66)^2(1.5) = 322 \text{ ft/s}^2 \quad \text{Ans.}
\]

16–23. The blade \( C \) of the power plane is driven by pulley \( A \) mounted on the armature shaft of the motor. If the constant angular acceleration of pulley \( A \) is \( \alpha_A = 40 \text{ rad/s}^2 \), determine the angular velocity of the blade at the instant \( A \) has turned 400 rev, starting from rest.

Motion of Pulley \( A \): Here, \( \theta_A = (400 \text{ rev}) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 800 \pi \text{ rad} \). Since the angular velocity can be determined from

\[
\omega_A^2 = (\omega_A)_0^2 + 2\alpha_C [\theta_A - (\theta_A)_0] \\
\omega_A^2 = 0^2 + 2(40)(800\pi - 0) \\
\omega_A = 448.39 \text{ rad/s}
\]

Motion of Pulley \( B \): Since blade \( C \) and pulley \( B \) are on the same axle, both will have the same angular velocity. Pulley \( B \) is connected to pulley \( A \) by a nonslip belt. Thus,

\[\omega_B r_B = \omega_A r_A\]

\[\omega_C = \omega_B = \left( \frac{r_A}{r_B} \right) \omega_A = \left( \frac{25}{50} \right) (448.39) = 224 \text{ rad/s} \quad \text{Ans.}\]
16–67. The bicycle has a velocity $v = 4$ ft/s, and at the same instant the rear wheel has a clockwise angular velocity $\omega = 3$ rad/s, which causes it to slip at its contact point A. Determine the velocity of point A.

$v_A = v_C + v_{A/C}$

\[
\begin{bmatrix}
  v_A \\
  v_C
\end{bmatrix} = \begin{bmatrix}
  4 \\
  0
\end{bmatrix} + \left( \begin{bmatrix}
  12 \\
  26
\end{bmatrix} \times \begin{bmatrix}
  0 \\
  3
\end{bmatrix} \right)
\]

$v_A = 2.5$ ft/s

Also,

$v_A = v_C + \omega \times r_{A/C}$

$v_A = 4i - 3k \times \left( \begin{bmatrix}
  12 \\
  26
\end{bmatrix} \right)$

$v_A = 4i - 6.5i = -2.5i$

$v_A = 2.5$ ft/s

*16–68. If bar $AB$ has an angular velocity $\omega_{AB} = 4$ rad/s, determine the velocity of the slider block $C$ at the instant shown.

For link $AB$: Link $AB$ rotates about a fixed point $A$. Hence

$v_B = \omega_{AB} r_{AB} = 4(0.15) = 0.6$ m/s

For link $BC$

$v_B = [0.6 \cos 30^\circ i - 0.6 \sin 30^\circ j] \text{m/s}$

$v_C = v_C i \quad \omega = \omega_{BC} k$

$r_{C/B} = [-0.2 \sin 30^\circ i + 0.2 \cos 30^\circ j] \text{m}$

$v_C = v_B + \omega \times r_{C/B}$

$v_C i = (0.6 \cos 30^\circ i - 0.6 \sin 30^\circ j) + (\omega_{BC} k) \times (-0.2 \sin 30^\circ i + 0.2 \cos 30^\circ j)$

$v_C i = (0.5196 - 0.1732\omega_{BC})i - (0.3 + 0.1\omega_{BC})j$

Equating the i and j components yields:

$0 = 0.3 + 0.1\omega_{BC} \quad \omega_{BC} = -3 \text{ rad/s}$

$v_C = 0.5196 - 0.1732(-3) = 1.04$ m/s →

Ans.
If link \( AB \) has an angular velocity of \( \omega_{AB} = 4 \text{ rad/s} \) at the instant shown, determine the velocity of the slider block \( E \) at this instant. Also, identify the type of motion of each of the four links.

**Link \( AB \)**

Rotates about the fixed point \( A \). Hence

\[
v_B = \omega_{AB} r_{AB} = 4(2) = 8 \text{ ft/s}
\]

For link \( BD \)

\[
v_B = [-8 \cos 60^\circ \mathbf{i} - 8 \sin 60^\circ \mathbf{j}] \text{ ft/s} \quad v_D = -v_D \mathbf{i} \quad \omega_{BD} = \omega_{BD} \mathbf{k}
\]

\[
r_{D/B} = [1 \mathbf{i}] \text{ ft}
\]

\[
v_D = v_B + \omega_{BD} \times r_{D/B}
\]

\[
-\mathbf{v}_D \mathbf{i} = (-8 \cos 60^\circ \mathbf{i} - 8 \sin 60^\circ \mathbf{j}) + (\omega_{BD} \mathbf{k}) \times (1 \mathbf{i})
\]

\[
-\mathbf{v}_D \mathbf{i} = -8 \cos 60^\circ \mathbf{i} + (\omega_{BD} - 8 \sin 60^\circ) \mathbf{j}
\]

\[
(\downarrow) \quad -\mathbf{v}_D = -8 \cos 60^\circ \mathbf{i} \quad v_D = 4 \text{ ft/s}
\]

\[
(+) \quad 0 = \omega_{BD} - 8 \sin 60^\circ \quad \omega_{BD} = 6.928 \text{ rad/s}
\]

For Link \( DE \)

\[
v_D = [-4 \mathbf{i}] \text{ ft/s} \quad \omega_{DE} = \omega_{DE} \mathbf{k} \quad \mathbf{v}_E = -v_E \mathbf{i}
\]

\[
r_{E/D} = [2 \cos 30^\circ \mathbf{i} + 2 \sin 30^\circ \mathbf{j}] \text{ ft}
\]

\[
v_E = v_D + \omega_{DE} \times r_{E/D}
\]

\[
-\mathbf{v}_E \mathbf{i} = -4 \mathbf{i} + (\omega_{DE} \mathbf{k}) \times (2 \cos 30^\circ \mathbf{i} + 2 \sin 30^\circ \mathbf{j})
\]

\[
-\mathbf{v}_E \mathbf{i} = (-4 - 2 \sin 30^\circ \omega_{DE}) \mathbf{i} + 2 \cos 30^\circ \omega_{DE} \mathbf{j}
\]

\[
(\downarrow) \quad 0 = 2 \cos 30^\circ \omega_{DE} \quad \omega_{DE} = 0
\]

\[
(+) \quad -\mathbf{v}_E = -4 - 2 \sin 30^\circ (0) \quad v_E = 4 \text{ ft/s}
\]

**Ans.**
*16–92. If end $A$ of the cord is pulled down with a velocity of $v_A = 4 \text{ m/s}$, determine the angular velocity of the spool and the velocity of point $C$ located on the outer rim of the spool.

**General Plane Motion:** Since the contact point $B$ between the rope and the spool is at rest, the IC is located at point $B$, Fig. a. From the geometry of Fig. a,

$$r_{A/IC} = 0.25 \text{ m}$$

$$r_{C/IC} = \sqrt{0.25^2 + 0.5^2} = 0.5590 \text{ m}$$

$$\phi = \tan^{-1}\left(\frac{0.25}{0.5}\right) = 26.57^\circ$$

Thus, the angular velocity of the spool can be determined from

$$\omega = \frac{v_A}{r_{A/IC}} = \frac{4}{0.25} = 16 \text{rad/s}$$

Then,

$$v_C = \omega r_{C/IC} = 16(0.5590) = 8.94 \text{m/s}$$

and its direction is

$$\theta = \phi = 26.6^\circ$$

---

•16–93. If end $A$ of the hydraulic cylinder is moving with a velocity of $v_A = 3 \text{ m/s}$, determine the angular velocity of rod $BC$ at the instant shown.

**Rotation About a Fixed Axis:** Referring to Fig. a,

$$v_B = \omega_{BC} r_B = \omega_{BC}(0.4)$$

**General Plane Motion:** The location of the IC for rod $AB$ is indicated in Fig. b. From the geometry shown in this figure, we obtain

$$r_{A/IC} = \frac{0.4}{\cos 45^\circ} = 0.5657 \text{ m}$$

$$r_{B/IC} = 0.4 \tan 45^\circ = 0.4 \text{ m}$$

Thus, the angular velocity of rod $AB$ can be determined from

$$\omega_{AB} = \frac{v_A}{r_{A/IC}} = \frac{3}{0.5657} = 5.303 \text{rad/s}$$

Then,

$$v_B = \omega_{AB} r_{B/IC}$$

$$\omega_{BC} (0.4) = 5.303(0.4)$$

$$\omega_{BC} = 5.30 \text{ rad/s}$$
At the given instant member $AB$ has the angular motions shown. Determine the velocity and acceleration of the slider block $C$ at this instant.

\[
v_B = 3(7) = 21 \text{ in./s}
\]

\[
v_C = v_B + \omega \times r_{C/B}
\]

\[
-v_C(\frac{4}{5})i - v_C(\frac{3}{5})j = -21i + \omega k \times (-5i - 12j)
\]

(↓) \quad -0.8v_C = -21 + 12\omega

(↑) \quad -0.6v_C = -5\omega

Solving:

\[
\omega = 1.125 \text{ rad/s}
\]

\[
v_C = 9.375 \text{ in./s} = 9.38 \text{ in./s}
\]

\[
(a_B)_n = (3)^2(7) = 63 \text{ in./s}^2
\]

\[
(a_B)_v = (2)(7) = 14 \text{ in./s}^2
\]

\[
a_C = a_B + \alpha \times r_{C/B} - \omega^2 r_{C/B}
\]

\[
-a_C(\frac{4}{5})i - a_C(\frac{3}{5})j = -14i - 63j + (\omega k) \times (-5i - 12j) - (1.125)^2(-5i - 12j)
\]

(↓) \quad -0.8a_C = -14 + 12\alpha + 6.328

(↑) \quad -0.6a_C = -63 - 5\alpha + 15.1875

\[
a_C = 54.7 \text{ in./s}^2
\]

\[
\alpha = -3.00 \text{ rad/s}^2
\]
16–142. At the instant shown rod $AB$ has an angular velocity $\omega_{AB} = 4 \text{ rad/s}$ and an angular acceleration $\alpha_{AB} = 2 \text{ rad/s}^2$. Determine the angular velocity and angular acceleration of rod $CD$ at this instant. The collar at $C$ is pin connected to $CD$ and slides freely along rod $AB$.

**Coordinate Axes:** The origin of both the fixed and moving frames of reference are located at point $A$. The $x, y, z$ moving frame is attached to and rotate with rod $AB$ since collar $C$ slides along rod $AB$.

**Kinematic Equation:** Applying Eqs. 16–24 and 16–27, we have

$$v_C = v_A + \Omega \times r_{C/A} + (v_{C/A})_{xyz} \quad [1]$$

$$a_C = a_A + \dot{\Omega} \times r_{C/A} + \Omega \times (\Omega \times r_{C/A}) + 2\Omega \times (v_{C/A})_{xyz} + (a_{C/A})_{xyz} \quad [2]$$

*Motion of moving reference*  
*Motion of $C$ with respect to moving reference*

$v_A = 0$

$a_A = 0$

$r_{C/A} = \{0.75i\} \text{ m}$

$\Omega = 4k \text{ rad/s}$

$(v_{C/A})_{xyz} = (v_{C/A})_{xyz} i$

$\dot{\Omega} = 2k \text{ rad/s}^2$

$(a_{C/A})_{xyz} = (a_{C/A})_{xyz} i$

The velocity and acceleration of collar $C$ can be determined using Eqs. 16–9 and 16–14 with $r_{C/D} = [-0.5 \cos 30^\circ i - 0.5 \sin 30^\circ j] \text{ m} = [-0.4330i - 0.250j] \text{ m}$.

$$v_C = \omega_{CD} \times r_{C/D} = -\omega_{CD} k \times (-0.4330i - 0.250j)$$

$$= -0.250 \omega_{CD} i + 0.4330 \omega_{CD} j$$

$$a_C = \alpha_{CD} \times r_{C/D} - \omega_{CD}^2 r_{C/D}$$

$$= -\alpha_{CD} k \times (-0.4330i - 0.250j) - \omega_{CD}^2 (-0.4330i - 0.250j)$$

$$= (0.4330 \alpha_{CD}^2 - 0.250 \alpha_{CD}) i + (0.4330 \alpha_{CD} + 0.250 \omega_{CD}^2) j$$

Substitute the above data into Eq. [1] yields

$$v_C = v_A + \Omega \times r_{C/A} + (v_{C/A})_{xyz}$$

$$-0.250 \omega_{CD} i + 0.4330 \omega_{CD} j = 0 + 4k \times 0.75i + (v_{C/A})_{xyz} i$$

$$-0.250 \omega_{CD} i + 0.4330 \omega_{CD} j = (v_{C/A})_{xyz} i + 3.00 j$$

Equating $i$ and $j$ components and solve, we have

$$(v_{C/A})_{xyz} = -1.732 \text{ m/s}$$

$$\omega_{CD} = 6.928 \text{ rad/s} = 6.93 \text{ rad/s} \quad \text{Ans.}$$

Substitute the above data into Eq. [2] yields

$$a_C = a_A + \dot{\Omega} \times r_{C/A} + \Omega \times (\Omega \times r_{C/A}) + 2\Omega \times (v_{C/A})_{xyz} + (a_{C/A})_{xyz}$$

$$= \dot{\Omega} \times r_{C/A} + \Omega \times (\Omega \times r_{C/A}) + 2\Omega \times (v_{C/A})_{xyz} + (a_{C/A})_{xyz}$$

$$= 0 + 4k \times 0.75i + 4k \times (4k \times 0.75i) + 2 \times (4k) \times (-1.732i) + (a_{C/A})_{xyz} i$$

$$= (20.78 - 0.250 \alpha_{CD} i) + (0.4330 \alpha_{CD} + 12) j$$

$$(a_{C/A})_{xyz} = 46.85 \text{ m/s}^2$$

$$\alpha_{CD} = -56.2 \text{ rad/s}^2 = 56.2 \text{ rad/s}^2 \quad \text{Ans.}$$
16–159. The quick return mechanism consists of the crank CD and the slotted arm AB. If the crank rotates with the angular velocity and angular acceleration at the instant shown, determine the angular velocity and angular acceleration of AB at this instant.

Reference Frame: The $xyz$ rotating reference frame is attached to slotted arm AB and coincides with the $XYZ$ fixed reference frame at the instant considered, Fig. a. Thus, the motion of the $xyz$ reference frame with respect to the $XYZ$ frame is

$$v_A = a_A = 0 \quad \omega_{AB} = \omega_{AB} \mathbf{k} \quad \dot{\omega}_{AB} = \alpha_{AB} \mathbf{k}$$

For the motion of point D with respect to the $xyz$ frame, we have

$$r_{D/A} = [4i] \text{ ft} \quad (v_{rel})_{xyz} = (v_{rel})_{xyz} i + (a_{rel})_{xyz} \mathbf{i}$$

Since the crank CD rotates about a fixed axis, $v_D$ and $a_D$ with respect to the $XYZ$ reference frame can be determined from

$$v_D = \omega_{CD} \times r_D$$

$$= (6\mathbf{k}) \times (2 \cos 30^\circ \mathbf{i} - 2 \sin 30^\circ \mathbf{j})$$

$$= [6\mathbf{i} + 10.39\mathbf{j}] \text{ ft/s}$$

$$a_D = \alpha_{CD} \times r_D - \omega_{CD}^2 r_D$$

$$= (3\mathbf{k}) \times (2 \cos 30^\circ \mathbf{i} - 2 \sin 30^\circ \mathbf{j}) - 6^2(2 \cos 30^\circ \mathbf{i} - 2 \sin 30^\circ \mathbf{j})$$

$$= [-59.35\mathbf{i} + 41.20\mathbf{j}] \text{ ft/s}^2$$

Velocity: Applying the relative velocity equation,

$$v_D = v_A + \omega_{AB} \times r_{D/A} + (v_{rel})_{xyz}$$

$$6\mathbf{i} + 10.39\mathbf{j} = 0 + (\omega_{AB} \mathbf{k}) \times (4\mathbf{i}) + (v_{rel})_{xyz} \mathbf{i}$$

$$6\mathbf{i} + 10.39\mathbf{j} = (v_{rel})_{xyz} \mathbf{i} + 4\omega_{AB} \mathbf{j}$$

Equating the i and j components yields

$$(v_{rel})_{xyz} = 6 \text{ ft/s}$$

$$10.39 = 4\omega_{AB} \quad \omega_{AB} = 2.598 \text{ rad/s} = 2.60 \text{ rad/s} \quad \text{Ans.}$$

Acceleration: Applying the relative acceleration equation,

$$a_D = a_A + \dot{\omega}_{AB} \times r_{D/A} + \omega_{AB} \times (\omega_{AB} \times r_{AB}) + 2\omega_{AB} \times (v_{rel})_{xyz} + (a_{rel})_{xyz}$$

$$-59.35\mathbf{i} + 41.20\mathbf{j} = 0 + (\alpha_{AB} \mathbf{k}) \times 4\mathbf{i} + 2.598\mathbf{k} \times [(2.598\mathbf{k}) \times (4\mathbf{i})] + 2(2.598\mathbf{k}) \times (6\mathbf{i}) + (a_{rel})_{xyz} \mathbf{i}$$

$$-59.35\mathbf{i} + 41.20\mathbf{j} = [(a_{rel})_{xyz} - 27] \mathbf{i} + (4\alpha_{AB} + 31.18) \mathbf{j}$$

Equating the i and j components yields

$$41.20 = 4\alpha_{AB} + 31.18 \quad \alpha_{AB} = 2.50 \text{ rad/s}^2 \quad \text{Ans.}$$