ME 205

DYNAMICS

HOMEWORK ASSIGNMENT # 6


SOLUTION

FALL 2006
Problem 14-2

The crate of weight \( W \) has a velocity \( v_A \) when it is at \( A \). Determine its velocity after it slides down the plane to \( s = s' \). The coefficient of kinetic friction between the crate and the plane is \( \mu_k \).

Given:

\[
W = 20 \text{ lb} \quad a = 3
\]
\[
v_A = 12 \text{ ft/s} \quad b = 4
\]
\[
s' = 6 \text{ ft}
\]
\[
\mu_k = 0.2
\]

Solution:

\[
\theta = \tan^{-1}\left(\frac{a}{b}\right) \quad N_C = W \cos(\theta) \quad F = \mu_k N_C
\]

Guess \( v' = 1 \text{ m/s} \)

Given \[\frac{1}{2} \left( \frac{W}{g} \right) v_A^2 + W \sin(\theta) s' - F s' = \frac{1}{2} \left( \frac{W}{g} \right) v'^2 \]

\( v' = \text{Find}(v') \quad v' = 17.72 \text{ ft/s} \)

Problem 14-10

The ball of mass \( M \) of negligible size is fired up the vertical circular track using the spring plunger. The plunger keeps the spring compressed a distance \( \delta \) when \( x = 0 \). Determine how far \( x \) it must be pulled back and released so that the ball will begin to leave the track when \( \theta = \theta_f \).

Given:

\[
M = 0.5 \text{ kg}
\]
\[
\delta = 0.08 \text{ m}
\]
\[
\theta_f = 135 \text{ deg}
\]
\[
r = 1.5 \text{ m}
\]
\[
k = 500 \text{ N/m}
\]
\[
g = 9.81 \text{ m/s}^2
\]

Solution:

\[
N = 0 \quad \theta = \theta_f
\]

\[
\Sigma F_n = ma_n \quad N - Mg \cos(\theta) = M \left( \frac{v^2}{r} \right) \quad v = \sqrt{gr \cos(\theta)} \quad v = 3.226 \text{ m/s}
\]
Problem 14-14

Determine the velocity of the block $A$ of weight $W_A$ if the two blocks are released from rest and the block $B$ of weight $W_B$ moves a distance $d$ up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is $\mu_k$.

Given:

\[
\begin{align*}
W_A &= 60 \text{ lb} \\
W_B &= 40 \text{ lb} \\
\theta_1 &= 60 \text{ deg} \\
\theta_2 &= 30 \text{ deg} \\
d &= 2 \text{ ft} \\
\mu_k &= 0.10
\end{align*}
\]

Solution:

\[
\begin{align*}
L &= 2s_A + s_B \\
0 &= 2v_A + v_B
\end{align*}
\]

Guesses

\[
\begin{align*}
v_A &= 1 \text{ ft/s} \\
v_B &= -1 \text{ ft/s}
\end{align*}
\]

Given

\[
\begin{align*}
0 &= 2v_A + v_B \\
W_A \left( \frac{d}{2} \right) \sin(\theta_1) - W_B d \sin(\theta_2) - \mu_k W_A \cos(\theta_1) \frac{d}{2} &= \frac{1}{2g} \left( W_A v_A^2 + W_B v_B^2 \right) \\
-\mu_k W_B \cos(\theta_2) d &= \left( \begin{array}{c} v_A \\ v_B \end{array} \right) = \text{Find}(v_A, v_B) \\
v_B &= -1.543 \text{ ft/s} \\
v_A &= 0.771 \text{ ft/s}
\end{align*}
\]
Problem 14-17

The block of weight $W$ slides down the inclined plane for which the coefficient of kinetic friction is $\mu_k$. If it is moving at speed $v$ when it reaches point $A$, determine the maximum deformation of the spring needed to momentarily arrest the motion.

Given:

$W = 100 \text{ lb}$  
$a = 3 \text{ m}$  
$v = 10 \frac{\text{ ft}}{\text{s}}$  
$b = 4 \text{ m}$  
$d = 10 \text{ ft}$  
$k = 200 \frac{\text{ lb}}{\text{ ft}}$  
$\mu_k = 0.25$

Solution:

$$N = \left(\frac{b}{\sqrt{a^2 + b^2}}\right)W$$

$N = 80 \text{ lb}$

Initial Guess

$$d_{\max} = 5 \text{ m}$$

*Problem 14-24

The block has a mass $M$ and moves within the smooth vertical slot. If it starts from rest when the attached spring is in the unstretched position at $A$, determine the constant vertical force $F$ which must be applied to the cord so that the block attains a speed $v_B$ when it reaches $s_B$.

Neglect the size and mass of the pulley. Hint: The work of $F$ can be determined by finding the difference $\Delta l$ in cord lengths $AC$ and $BC$ and using $U_F = F \Delta l$.

Given:

$M = 0.8 \text{ kg}$  
$l = 0.4 \text{ m}$  
$v_B = 2.5 \frac{\text{ m}}{\text{s}}$  
$b = 0.3 \text{ m}$
\[ s_B = 0.15 \text{ m} \quad k = 100 \frac{\text{N}}{\text{m}} \]

Solution:
\[ \Delta l = \sqrt{l^2 + b^2} - \sqrt{(l - s_B)^2 + b^2} \]

Guess \( F = 1 \text{ N} \)

Given
\[ F \Delta l - Mgs_B - \frac{1}{2}ks_B^2 = \frac{1}{2}Mv_B^2 \]

\( F = \text{Find}(F) \quad F = 43.9 \text{ N} \)

*Problem 14-44*

A truck has a weight \( W \) and an engine which transmits a power \( P \) to all the wheels. Assuming that the wheels do not slip on the ground, determine the angle \( \theta \) of the largest incline the truck can climb at a constant speed \( v \).

Given:
\[ W = 25000 \text{ lbf} \]
\[ v = 50 \text{ ft} \text{ s}^{-1} \]
\[ P = 350 \text{ hp} \]

Solution:
\[ F = W \sin(\theta) \quad P = W \sin(\theta)v \]
\[ \theta = \sin \left( \frac{P}{Wv} \right) \quad \theta = 8.86 \text{ deg} \]
Problem 14-54

The crate has mass $m_c$ and rests on a surface for which the coefficients of static and kinetic friction are $\mu_s$ and $\mu_k$ respectively. If the motor $M$ supplies a cable force of $F = at^2 + b$, determine the power output developed by the motor when $t = t_f$.

Given:

\[
\begin{align*}
    m_c &= 150 \text{ kg} \\
    a &= 8 \frac{\text{N}}{\text{s}^2} \\
    \mu_s &= 0.3 \\
    b &= 20 \text{ N} \\
    \mu_k &= 0.2 \\
    t_f &= 5 \text{ s} \\
    g &= 9.81 \frac{\text{m}}{\text{s}^2}
\end{align*}
\]

Solution:

Time to start motion

\[
3(at_f^2 + b) = \mu_s m_c g \\
t_f = \frac{1}{a} \left( \frac{\mu_s m_c g}{3} - b \right) \\
t_f = 3.99 \text{ s}
\]

Speed at $t_f$

\[
\begin{align*}
    3(at^2 + b) - \mu_k m_c g &= m_c a = m_c \frac{dv}{dt} \\
    v &= \int_{t}^{t_f} \frac{3}{m_c} (at^2 + b) - \mu_k g \, dt \\
    v &= 1.70 \frac{\text{m}}{\text{s}}
\end{align*}
\]

\[
\begin{align*}
    P &= 3(at_f^2 + b) v \\
    P &= 1.12 \text{ kW}
\end{align*}
\]

*Problem 14-60

The collar of weight $W$ starts from rest at $A$ and is lifted by applying a constant vertical force $F$ to the cord. If the rod is smooth, determine the power developed by the force at the instant $\theta = \theta_2$.

Given:

\[
\begin{align*}
    W &= 10 \text{ lbf} \\
    a &= 3 \text{ ft} \\
    F &= 25 \text{ lbf} \\
    b &= 4 \text{ ft} \\
    \theta_2 &= 60 \text{ deg}
\end{align*}
\]

Solution:

\[
h = b - (a) \cot(\theta_2)
\]

\[
\begin{align*}
    L_1 &= \sqrt{a^2 + b^2} \\
    L_2 &= \sqrt{a^2 + (b - h)^2} \\
    F(L_1 - L_2) - Wh &= \frac{1}{2} \left( \frac{W}{g} \right) v_2^2 \\
    v_2 &= \sqrt{2 \left( \frac{F}{W} \right) (L_1 - L_2) g - 2gh} \\
    P &= F v_2 \cos(\theta_2) \\
    P &= 0.229 \text{ hp}
\end{align*}
\]
Problem 14-71

If the spring is compressed a distance \( \delta \) against the block of weight \( W \) and it is released from rest, determine the normal force of the smooth surface on the block when it reaches the point \( x_f \).

**Given:**

\[
W = 0.5 \text{ lb} \\
b = 1 \text{ ft} \\
k = 5 \text{ lb/in} \\
\delta = 3 \text{ in} \\
x_f = 0.5 \text{ ft}
\]

**Solution:**

\[
y(x) = \frac{x^2}{2b} \quad y'(x) = \frac{x}{b} \quad y''(x) = \frac{1}{b} \quad \rho(x) = \frac{\sqrt{\left[1 + y'(x)^2\right]^3}}{y''(x)} \\
\theta(x) = \arctan(y'(x))
\]

\[
T_I = 0 \quad V_I = \frac{1}{2}k\delta^2 \quad T_I = \frac{1}{2} \left( \frac{W}{g} \right) v_I^2
\]

\[
0 + \frac{1}{2}k\delta^2 = \frac{1}{2} \left( \frac{W}{g} \right) v_I^2 + W y(x_f) \\
v_I = \sqrt{\left(k\delta^2 - 2 \ddot{V}_2 = W y''(x_f)\right)}
\]

\[
F_N - W \cos(\theta(x_f)) = \frac{W}{g} \left( \frac{v_I^2}{\rho(x_f)} \right)
\]

\[
F_N = W \cos(\theta(x_f)) + \frac{W}{g} \left( \frac{v_I^2}{\rho(x_f)} \right) \quad F_N = 3.041 \text{ lb}
\]

---

Problem 14-75

The bob of the pendulum has a mass \( M \) and is released from rest when it is in the horizontal position shown. Determine its speed and the tension in the cord at the instant the bob passes through its lowest position.

**Given:**

\[
M = 0.2 \text{ kg} \\
r = 0.75 \text{ m} \\
g = 9.81 \text{ m/s}^2
\]
Solution:

Datum at initial position:

\[ T_1 + V_1 = T_2 + V_2 \]

\[ 0 + 0 = \frac{1}{2} M v_2^2 - M g r \]

\[ v_2 = \sqrt{2gr} \quad v_2 = 3.84 \, \text{m/s} \]

\[ \Sigma F_n = Ma_n \]

\[ T - Mg = M \left( \frac{v_2^2}{r} \right) \]

\[ T = M \left( g + \frac{v_2^2}{r} \right) \quad T = 5.89 \, \text{N} \]

*Problem 14-76*

The collar of weight \( W \) is released from rest at \( A \) and travels along the smooth guide. Determine the speed of the collar just before it strikes the stop at \( B \). The spring has an unstretched length \( L \).

Given:

\[ W = 5 \, \text{lb} \quad k = 2 \, \text{lb/in} \]

\[ L = 12 \, \text{in} \quad g = 32.2 \, \text{ft/s}^2 \]

\[ h = 10 \, \text{in} \]

Solution:

\[ T_A + V_A = T_B + V_B \]

\[ 0 + W(L + h) + \frac{1}{2} k h^2 = \frac{1}{2} \left( \frac{W}{g} \right) v_B^2 \]

\[ v_B = \sqrt{\left( \frac{kg}{W} \right) h^2 + 2g(L + h)} \quad v_B = 15.013 \, \text{ft/s} \]
Problem 14-81

The bob of mass $M$ of a pendulum is fired from rest at position $A$ by a spring which has a stiffness $k$ and is compressed a distance $\delta$. Determine the speed of the bob and the tension in the cord when the bob is at positions $B$ and $C$. Point $B$ is located on the path where the radius of curvature is still $r$, i.e., just before the cord becomes horizontal.

Units Used: $\text{kN} = 10^3 \text{ N}$

Given:

$M = 0.75 \text{ kg}$

$k = 6 \text{ kN/m}$

$\delta = 125 \text{ mm}$

$r = 0.6 \text{ m}$

Solution:

At $B$:

$0 + \frac{1}{2} k \delta^2 = \frac{1}{2} M v_B^2 + M g r$

$v_B = \sqrt{\left(\frac{k}{M}\right) \delta^2 - 2 g r}$

$v_B = 10.6 \text{ m/s}$

$T_B = M \left(\frac{v_B^2}{r}\right)$

$T_B = 142 \text{ N}$

At $C$:

$0 + \frac{1}{2} k \delta^2 = \frac{1}{2} M v_C^2 + M g 3r$

$v_C = \sqrt{\left(\frac{k}{M}\right) \delta^2 - 6 g r}$

$v_C = 9.47 \text{ m/s}$

$T_C + M g = M \left(\frac{v_C^2}{2r}\right)$

$T_C = M \left(\frac{v_C^2}{2r} - g\right)$

$T_C = 48.7 \text{ N}$