ME 205
DYNAMICS

HOMEWORK ASSIGNMENT # 4

SOLUTION
(13.2 - 13.81)

FALL 2006
13-2. The 10-lb block has an initial velocity of 10 ft/s on the smooth plane. If a force $F = (2.5t)$ lb, where $t$ is in seconds, acts on the block for 3 s, determine the final velocity of the block and the distance the block travels during this time.

\[ \Sigma F = ma : \quad 2.5t = \left( \frac{10}{32.2} \right) a \]

\[ a = 8.05t \]

\[ dv = a \, dt \]

\[ \int_{0}^{10} dv = \int_{0}^{10} 8.05t \, dt \]

\[ v = 4.025t^2 + 10 \]

When $t = 3$, \[ v = 46.2 \text{ ft/s} \quad \text{Ans} \]

\[ \frac{ds}{dt} = v \]

\[ \int_{0}^{s} \, ds = \int_{0}^{s} \left( 4.025t^2 + 10 \right) \, dt \]

\[ s = 1.3417t^3 + 10t \]

When $t = 3$, \[ s = 66.2 \text{ ft} \quad \text{Ans} \]

13-6. The baggage truck $A$ has a mass of 800 kg and is used to pull the two cars, each with mass 300 kg. If the tractive force $F$ on the truck is $F = 480$ N, determine the initial acceleration of the truck. What is the acceleration of the truck if the coupling at $C$ suddenly fails? The car wheels are free to roll. Neglect the mass of the wheels.

\[ \Sigma F = ma : \quad 480 = (800 + 2(300))a \]

\[ a = 0.3429 = 0.343 \text{ m/s}^2 \quad \text{Ans} \]

\[ \Sigma F = ma : \quad 480 = (800 + 300)a \]

\[ a = 0.436 \text{ m/s}^2 \quad \text{Ans} \]
13-12. The 6-lb particle is subjected to the action of its weight and forces \( F_1 = (2i + 6j - 2k) \) lb, \( F_2 = (i - 4j - 1k) \) lb, and \( F_3 = (-2i) \) lb, where \( t \) is in seconds. Determine the distance the ball is from the origin 2 s after being released from rest.

\[ \Sigma F = ma. \quad (2i + 6j - 2k) + (i - 4j - 1k) - 2d - 6k = \left( \frac{6}{32.2} \right) (a_x i + a_y j + a_z k) \]

Equating components:

\[ \left( \frac{6}{32.2} \right) a_x = t^2 - 2t + 2 \quad \left( \frac{6}{32.2} \right) a_y = -4t + 6 \quad \left( \frac{6}{32.2} \right) a_z = -2t - 7 \]

Since \( dv = adt \), integrating from \( v = 0 \), \( t = 0 \), yields:

\[ \left( \frac{6}{32.2} \right) v_x = \frac{t^2}{2} - t^2 + 2t \quad \left( \frac{6}{32.2} \right) v_y = -2t^2 + 6t \quad \left( \frac{6}{32.2} \right) v_z = -t^2 - 7t \]

Since \( ds = v dt \), integrating from \( s = 0 \), \( t = 0 \) yields:

\[ \left( \frac{6}{32.2} \right) s_x = \frac{t^3}{12} - \frac{t^3}{3} + t^2 \quad \left( \frac{6}{32.2} \right) s_y = \frac{-2t^3}{3} + 3t^2 \quad \left( \frac{6}{32.2} \right) s_z = \frac{-t^3}{3} - \frac{7t^2}{2} \]

When \( t = 2 \) s then, \( s_x = 14.31 \text{ ft} \), \( s_y = 35.78 \text{ ft} \), \( s_z = -89.44 \text{ ft} \)

Thus,

\[ s = \sqrt{(14.31)^2 + (35.78)^2 + (-89.44)^2} = 97.4 \text{ ft} \quad \text{Ans} \]

13-23. A force \( F = 15 \) lb is applied to the cord. Determine how high the 30-lb block \( A \) rises in 2 s starting from rest. Neglect the weight of the pulleys and cord.

\[ \Sigma F = ma. \quad -30 + 4F = \frac{30}{32.2} a_x \]

\[ F = 15 \text{ lb} \]

\[ a_x = \frac{32.2}{2} \text{ ft/s}^2 \]

\[ (+v) v = v_0 + v_0 t + \frac{1}{2} a_x t^2 \]

\[ v = 0 + 0 + \frac{1}{2} (32.2)(2)^2 \]

\[ s = 64.4 \text{ ft} \quad \text{Ans} \]
13-25. Determine the required mass of block $A$ so that when it is released from rest it moves the 5-kg block $B$ 0.75 m up along the smooth inclined plane in $t = 2$ s. Neglect the mass of the pulleys and cords.

**Kinematic:** Applying equation $s = s_0 + v_0t + \frac{1}{2}a_t t^2$, we have

$$s = 0 + 0 + \frac{1}{2}a_t (2)^2 \quad a_t = 0.375 \text{ m/s}^2$$

Establish the position-coordinate equation, we have

$$2s_A + (s_A - s_B) = 1 \quad 3s_A - s_B = 1$$

Taking time derivative twice yields

$$3a_A - a_B = 0 \quad [1]$$

From Eq [1],

$$3a_A - 0.375 = 0 \quad a_A = 0.125 \text{ m/s}^2$$

**Equation of Motion:** The tension $T$ developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys. From FBD(b),

$$+ \sum F_y = m_a a_y \quad T - 5(9.81) \sin 60° = 5(0.375) \quad T = 44.35 \text{ N}$$

From FBD(a),

$$+ \sum F_y = m_a a_y \quad 3(44.35) - 9.81 m_a = m_a (-0.125) \quad m_a = 13.7 \text{ kg} \quad \text{Ans}$$

13-33. The 2-kg collar $C$ is free to slide along the smooth shaft $AB$. Determine the acceleration of collar $C$ if collar $A$ is subjected to an upward acceleration of 4 m/s$^2$.

$$+ \sum F_y = m_a a_y \quad N \sin 45° = 2a_{C/AB} \sin 45°$$

$$N = 2a_{C/AB}$$

$$+ \sum F_y = m_a a_y \quad N \cos 45° - 19.62 = 2(4) - 2a_{C/AB} \cos 45°$$

$$a_{C/AB} = 9.76514$$

$$a_C = a_{AB} + a_{C/AB}$$

$$\begin{align*}
(a_C)_x &= 0 + 9.76514 \sin 45° = 6.905 \\
(a_C)_y &= 4 - 9.76514 \cos 45° = 2.905
\end{align*}$$

$$a_C = \sqrt{(6.905)^2 + (2.905)^2} = 7.49 \text{ m/s}^2 \quad \text{Ans}$$

$$\theta_C = \tan^{-1} \left( \frac{2.905}{6.905} \right) = 22.8^\circ \text{ Ans}$$
13.42. Blocks A and B each have a mass \( m \). Determine the largest horizontal force \( P \) which can be applied to \( B \) so that \( A \) will not move relative to \( B \). All surfaces are smooth.

Require
\[
\sum F_x = ma
\]
Block A:
\[
+ \sum F_x = 0; \quad N \cos \theta - mg = 0
\]
\[
- \sum F_y = ma; \quad N \sin \theta = ma
\]
\[
a = g \tan \theta
\]
Block B:
\[
- \sum F_x = ma; \quad P - N \sin \theta = ma
\]
\[
P - mg \tan \theta = mg \tan \theta
\]
\[
P = 2mg \tan \theta \quad \text{Ans}
\]

13.51. The block \( A \) has a mass \( m_a \) and rests on the pan \( B \), which has a mass \( m_B \). Both are supported by a spring having a stiffness \( k \) that is attached to the bottom of the pan and to the ground. Determine the distance \( d \) the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes unstretched.

For Equilibrium
\[
\sum F_x = ma; \quad F_x = (m_a + m_b)g
\]
\[
y = \frac{F_x}{k} = \frac{(m_a + m_b)g}{k}
\]
Block:
\[
+ \sum F_x = m_a; \quad -m_a g + N = m_a a
\]
Block and pan
\[
+ \sum F_x = m_a; \quad -(m_a + m_b)g + k(y + y) = (m_a + m_b) a
\]
Thus,
\[
-(m_a + m_b)g + k\left(\frac{(m_a + m_b)g + y}{k}\right) = (m_a + m_b) - \frac{m_a g + N}{m_a}
\]
Require \( y = d, N = 0 \)
\[
k d = -(m_a + m_b)g
\]
Since \( d \) is downward,
\[
d = \frac{(m_a + m_b)g}{k} \quad \text{Ans}
\]
13-57. The 600-kg wrecking ball is suspended from the crane by a cable having a negligible mass. If the ball has a speed \( v = 8 \text{ m/s} \) at the instant it is at its lowest point, \( \theta = 0^\circ \), determine the tension in the cable at this instant. Also, determine the angle \( \theta \) to which the ball swings before it stops.

\[
\begin{align*}
- \sum F_x &= m a_x: \quad T - 600(9.81) = 600(\frac{v^2}{12}) \\
T &= 9086 \text{ N} = 9.08 \text{ kN} \quad \text{Ans}
\end{align*}
\]

Set \( a_x(12 \, \text{m}) = v \, \text{d}v \)

\[
-9.81(12) \int_0^v \sin \theta \, \text{d}v = \frac{1}{2} v^2
\]

\[
-9.81(12)(-\cos \theta + 1) = -\frac{1}{2}(8)^2
\]

\[\theta = 43.3^\circ \quad \text{Ans}\]

13-62. Solve Prob. 13-61 if the speed of the acrobat's center of mass is increased from \( (v_x)_0 = 10 \text{ ft/s} \) at \( \theta = 0^\circ \) by a constant rate of \( \dot{v}_x = 0.5 \text{ ft/s}^2 \).

**Kinematics:** Applying equation \( v^2 = v_0^2 + 2a_x(x - x_0) \), we have

\[v^2 = 10^2 + 2(0.5)(15\theta - 0) = 100 + 15\theta\]

**Equation of Motion:** If the acrobat is about to fly off the chair, the normal reaction \( N = 0 \). Applying Eq. 13-8, we have

\[
\sum F_x = ma_x: \quad 150 \cos \theta = \frac{150}{32.2} \left( \frac{100 + 15\theta}{15} \right)
\]

Solving for \( \theta \) by trial and error,

\[\theta = 75.6^\circ \quad \text{Ans}\]