ME 205
DYNAMICS
HOMEWORK ASSIGNMENT # 3

SOLUTION
(12.143 – 12.207)

FALL 2006
12-143. A particle moves in the $x$-$y$ plane such that its position is defined by $\mathbf{r} = (2t\mathbf{i} + 4t^2\mathbf{j})$ ft, where $t$ is in seconds. Determine the radial and tangential components of the particle's velocity and acceleration when $t = 2$ s.

\[
\begin{align*}
\mathbf{r} &= 2t\mathbf{i} + 4t^2\mathbf{j} \\
v &= 21 + 8t \\
a &= 8 \\
\theta &= \tan^{-1}\left(\frac{16}{4}\right) = 75.964^\circ \\
v &= \sqrt{(2)^2 + (16)^2} = 16.1245 \text{ ft/s} \\
\phi &= \tan^{-1}\left(\frac{16}{2}\right) = 82.875^\circ \\
a &= 8 \text{ ft/s}^2 \\
\phi - \theta &= 6.9112^\circ \\
v_r &= 16.1245 \cos 6.9112^\circ = 16.0 \text{ ft/s} \\
v_\theta &= 16.1245 \sin 6.9112^\circ = 1.94 \text{ ft/s} \\
\delta &= 90^\circ - \theta = 14.036^\circ \\
a_r &= 8 \cos 14.036^\circ = 7.76 \text{ ft/s}^2 \\
a_\theta &= 8 \sin 14.036^\circ = 1.94 \text{ ft/s}^2
\end{align*}
\]

*12-144. A truck is traveling along the horizontal circular curve of radius $r = 60$ m with a constant speed $v = 20$ m/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line $r$ and the magnitude of the truck's acceleration.

\[
\begin{align*}
r &= 60 \\
r &= 0 \\
\theta &= 0 \\
v &= 20 \\
v_r &= \dot{r} = 0 \\
v_\theta &= r\dot{\theta} = 60 \dot{\theta} \\
v &= \sqrt{(v_r)^2 + (v_\theta)^2} \\
20 &= 60 \dot{\theta} \\
\dot{\theta} &= 0.333 \text{ rad/s} \quad \text{Ans} \\
a_r &= \dot{r} - r(\dot{\theta})^2 \\
&= 0 - 60(0.333)^2 \\
&= -6.67 \text{ m/s}^2 \\
a_\theta &= r\ddot{\theta} + 2r\dot{\theta} \\
&= 60 \ddot{\theta} \\
Since \\
v &= r\dot{\theta} \\
v &= r\dot{\theta} + r\ddot{\theta} \\
0 &= 0 + 60 \ddot{\theta} \\
\ddot{\theta} &= 0 \\
Thus, \\
a_\theta &= 0 \\
a &= a_r = 6.67 \text{ m/s}^2 \quad \text{Ans}
\]
12-147. The slotted link is pinned at O, and as a result of the constant angular velocity \( \dot{\theta} = 3 \text{ rad/s} \) it drives the peg P for a short distance along the spiral guide \( r = (0.4 \theta) \text{ m} \), where \( \theta \) is in radians. Determine the radial and transverse components of the velocity and acceleration of P at the instant \( \theta = \pi/3 \text{ rad} \).

\[
\begin{align*}
\dot{\theta} &= 3 \text{ rad/s} \\
r &= 0.4 \theta \\
\dot{r} &= 0.4 \dot{\theta} \\
\end{align*}
\]

At \( \theta = \pi/3 \):

\[
\begin{align*}
r &= 0.4189 \\
\dot{r} &= 0.4(3) = 1.20 \\
r &= 0.4(0) = 0 \\
\end{align*}
\]

\( v_r = r \ddot{\theta} = 1.20 \text{ m/s} \quad \text{Ans} \)

\( v_p = r \dot{\theta} = 0.4189(3) = 1.26 \text{ m/s} \quad \text{Ans} \)

\( a_r = \ddot{r} - r \ddot{\theta} = 0 - 0.4189(3)^2 = -3.77 \text{ m/s}^2 \quad \text{Ans} \)

\( a_p = r \dddot{\theta} + 2 r \dot{\theta} \dot{\theta} = 0 + 2(1.20)(3) = 7.20 \text{ m/s}^2 \quad \text{Ans} \)

*12-148. Solve Prob. 12-147 if the slotted link has an angular acceleration \( \ddot{\theta} = 8 \text{ rad/s}^2 \) when \( \dot{\theta} = 3 \text{ rad/s} \) at \( \theta = \pi/3 \text{ rad} \).

\[
\begin{align*}
\dot{\theta} &= 3 \text{ rad/s} \\
r &= 0.4 \theta \\
\dot{r} &= 0.4 \dot{\theta} \\
\end{align*}
\]

\( \theta = \pi/3 \quad \dot{\theta} = 3 \quad \ddot{\theta} = 8 \)

\( r = 0.4189 \)

\( r = 1.20 \)

\( \dot{r} = 0.4(8) = 3.20 \)

\( v_r = r \ddot{\theta} = 1.20 \text{ m/s} \quad \text{Ans} \)

\( v_p = r \dot{\theta} = 0.4189(3) = 1.26 \text{ m/s} \quad \text{Ans} \)

\( a_r = \ddot{r} - r \dddot{\theta} = 3.20 - 0.4189(3)^2 = -0.570 \text{ m/s}^2 \quad \text{Ans} \)

\( a_p = r \dddot{\theta} + 2 r \dot{\theta} \dot{\theta} = 0.4189(8) + 2(1.20)(3) = 10.6 \text{ m/s}^2 \quad \text{Ans} \)
**12-168.** The pin follows the path described by the equation \( r = (0.2 + 0.15 \cos \theta) \) m. At the instant \( \theta = 30^\circ \), \( \dot{\theta} = 0.7 \) rad/s and \( \ddot{\theta} = 0.5 \) rad/s\(^2\). Determine the magnitudes of the pin's velocity and acceleration at this instant. Neglect the size of the pin.

\[
r = 0.2 + 0.15 \cos \theta = 0.2 + 0.15 \cos 30^\circ = 0.3299 \text{ m}
\]

\[
r = 0.15 \sin \dot{\theta} = -0.15 \sin 30^\circ (0.7) = -0.0525 \text{ m/s}
\]

\[
\ddot{r} = -0.15 \left[ \cos \dot{\theta} \ddot{\theta} + \sin \dot{\theta} \dot{\theta} \right] = -0.15 \left[ \cos 30^\circ (0.7) \ddot{\theta} + \sin 30^\circ (0.5) \right] = -0.10115 \text{ m/s}^2
\]

\[
u = \dot{r} = -0.0525 \text{ m/s}
\]

\[
v = \sqrt{\dot{r}^2 + \dot{\theta}^2} = \sqrt{(-0.0525)^2 + (0.2309)^2} = 0.237 \text{ m/s}
\]

\[
\begin{aligned}
a_r &= \ddot{r} - r \ddot{\theta}^2 = -0.10115 - 0.3299 (0.7)^2 = -0.2628 \text{ m/s}^2 \\
a_{\theta} &= -2 \dot{r} \dot{\theta} = 0.3299 (0.5) + 2 (-0.0525) (0.7) = 0.09145 \text{ m/s}^2 \\
a &= \sqrt{a_r^2 + a_{\theta}^2} = \sqrt{(-0.2628)^2 + (0.09145)^2} = 0.278 \text{ m/s}^2
\end{aligned}
\]

**12-172.** If the end of the cable at \( A \) is pulled down with a speed of 2 m/s, determine the speed at which block \( B \) rises.

**Position - Coordinate Equation:** Datum is established at fixed pulley \( C \). The position of point \( A \) and block \( B \) with respect to datum are \( s_A \) and \( s_B \), respectively.

\[
2s_A + s_B = l
\]

**Time Derivative:** Taking the time derivative of the above equation yields

\[
2v_B + v_A = 0 \quad (1)
\]

Since \( v_A = 5 \text{ m/s} \), from Eq. (1)

\[
2v_B + 2 = 0 \\
v_B = -1 \text{ m/s} = 1 \text{ m/s} \uparrow \quad \text{Ans}
\]

**12-178.** Determine the displacement of the block at \( B \) if \( A \) is pulled down 4 ft.

\[
2s_A + 2s_C = h
\]

\[
\Delta x_A = -\Delta x_C
\]

\[
s_A - s_C + s_B = h
\]

\[
2\Delta s_B = \Delta s_C
\]

Thus,

\[
2\Delta s_B = -\Delta s_A
\]

\[
2\Delta s_B = -4
\]

\[
\Delta s_B = -2 \text{ ft} = 2 \text{ ft} \uparrow \quad \text{Ans}
\]
12-181. If block $A$ is moving downward with a speed of 4 ft/s while $C$ is moving up at 2 ft/s, determine the speed of block $B$.

$$x_A = 2x_B + x_C = l$$

$$v_A = 2v_B + v_C = 0$$

$$4 + 2v_B - 2 = 0$$

$$v_B = -1 \text{ ft/s} = 1 \text{ ft/s} \quad \text{Ans}$$

12-188. The roller at $A$ is moving upward with a velocity of $v_A = 3 \text{ ft/s}$ and has an acceleration of $a_A = 4 \text{ ft/s}^2$ when $s_A = 4 \text{ ft}$. Determine the velocity and acceleration of block $B$ at this instant.

$$s_A + \sqrt{\left(s_A\right)^2 + 3^2} = l$$

$$v_A = \frac{1}{2} \left( \left(s_A\right)^2 + 3^2 \right)^{-1} \left( 2s_A \right) \hat{s}_A = 0$$

$$s_A = \left[ \left( s_A \right)^2 + 9 \right]^{-1} \left( s_A \hat{s}_A \right) = 0$$

$$v_A - \left[ \left( s_A \right)^2 + 9 \right]^{-1} \left( 2s_A \hat{s}_A \right) - \left[ \left( s_A + 9 \right)^2 + \left( s_A + 9 \right) \hat{s}_A \hat{s}_A \right] = 0$$

At $s_A = 4 \text{ ft}, \hat{s}_A = 3 \text{ ft/s}, \quad s_A = 4 \text{ ft/s}^2$

$$v_A = \left( \frac{1}{5} \right) \left( 4 \right) \left( 3 \right) = 0$$

$$v_B = -2.4 \text{ ft/s} = 2.40 \text{ ft/s} \quad \text{Ans}$$

$$v_B - \left( \frac{1}{5} \right) \left( 4 \right)^2 \left( 3 \right)^2 + \left( \frac{1}{5} \right) \left( 3 \right)^2 + \left( \frac{1}{5} \right) \left( 4 \right) \left( 4 \right) = 0$$

$$a_B = -3.85 \text{ ft/s}^2 = 3.85 \text{ ft/s}^2 \quad \text{Ans}$$
12-197. At the instant shown, cars $A$ and $B$ are traveling at speeds of 30 mi/h and 20 mi/h, respectively. If $B$ is increasing its speed by 1200 mi/h$^2$, while $A$ maintains a constant speed, determine the velocity and acceleration of $B$ with respect to $A$.

\[
\begin{align*}
\dot{v}_B &= 20 \text{ mi/h} \\
v_A &= 30 \text{ mi/h} \\
\end{align*}
\]

\[v_B = v_A + v_{BA}
\]

\[20 \sqrt{30^2 + 0.3^2} = 30 + (v_{BA})_x + (v_{BA})_y
\]

\([-\cos 30^\circ] - 20 \sin 30^\circ = -30 + (v_{BA})_x
\]

\([+ \tan 30^\circ] 20 \cos 30^\circ = (v_{BA})_y
\]

Solving

\[(v_{BA})_x = 20 \rightarrow\]

\[(v_{BA})_y = 17.32 \uparrow\]

\[v_{BA} = \sqrt{(20)^2 + (17.32)^2} = 26.5 \text{ mi/h} \quad \text{Ans}
\]

\[\theta = \tan^{-1} \frac{17.32}{20} = 40.5^\circ \quad \text{Ans}
\]

\[a_B = a_A + a_{BA}
\]

\[1200 \sqrt{30^2 + 1333.3^2} \quad \sqrt{30^2} \quad \theta = 0 + (a_{BA})_x + (a_{BA})_y
\]

\([-\cos 30^\circ] - 1200 \sin 30^\circ + 1333.3 \cos 30^\circ = (a_{BA})_x
\]

\([+ \tan 30^\circ] 1200 \cos 30^\circ + 1333.3 \sin 30^\circ = (a_{BA})_y
\]

Solving

\[(a_{BA})_x = 554.7 \rightarrow \quad (a_{BA})_y = 1705.9 \uparrow \quad \text{Ans}
\]

\[a_{BA} = \sqrt{(554.7)^2 + (1705.9)^2} = 1.79 \times 10^3 \text{ mi/h}^2 \quad \text{Ans}
\]

\[\theta = \tan^{-1} \frac{1705.9}{554.7} = 72.9^\circ \downarrow\quad \text{Ans}
\]

12-198. At the instant shown, cars $A$ and $B$ are traveling at speeds of 30 mi/h and 20 mi/h, respectively. If $A$ is increasing its speed at 400 mi/h$^2$ whereas the speed of $B$ is decreasing at 800 mi/h$^2$, determine the velocity and acceleration of $B$ with respect to $A$.

\[
\begin{align*}
\dot{v}_B &= 20 \text{ mi/h} \\
v_A &= 30 \text{ mi/h} \\
\end{align*}
\]

\[a_B = a_A + a_{BA}
\]

\[20 \sqrt{30^2 + 0.3^2} = 30 + (v_{BA})_x + (v_{BA})_y
\]

\([-\cos 30^\circ] - 400 \sin 30^\circ = -30 + (v_{BA})_x
\]

\([+ \tan 30^\circ] 20 \cos 30^\circ = (v_{BA})_y
\]

Solving

\[(v_{BA})_x = 1533 \rightarrow \quad (v_{BA})_y = 1533 \uparrow \quad \text{Ans}
\]

\[(a_{BA})_x = -26.154 \downarrow \quad (a_{BA})_y = -26.154 \uparrow
\]

\[a_{BA} = \sqrt{(-26.154)^2 + (26.154)^2} = 555 \text{ mi/h}^2 \quad \text{Ans}
\]

\[\theta = \tan^{-1} \frac{26.154}{-26.154} = 0.767^\circ \quad \text{Ans}
\]
12-207. At a given instant the football player at A throws a football C with a velocity of 20 m/s in the direction shown. Determine the constant speed at which the player at B must run so that he can catch the football at the same elevation at which it was thrown. Also calculate the relative velocity and relative acceleration of the football with respect to B at the instant the catch is made. Player B is 15 m away from A when A starts to throw the football.

![Diagram of football throw](image_url)

**Ball:**

1. \( \vec{v}_B = \vec{v}_C + \vec{v}_A \)
2. \( \vec{a}_C = 0 + 20 \cos 60^\circ \hat{t} \)
3. \( \vec{v} = \vec{v}_B + \vec{v}_A \)
4. \( -20 \sin 60^\circ = 20 \sin 60^\circ - 9.81 \hat{t} \)
5. \( \hat{t} = 3.53 \text{ s} \)
6. \( \vec{x}_C = 35.31 \text{ m} \)

**Player B:**

1. \( \vec{x}_B = \vec{x}_A + \vec{v}_B t \)
2. \( \vec{x}_A = 0 + \vec{v}_A t \)

Require:

\( 35.31 = 15 + v_B (3.33) \)

\( v_B = 5.75 \text{ m/s} \quad \text{Ans} \)

At the time of the catch:

1. \( \vec{v}_{Cz} = 20 \cos 60^\circ = 10 \text{ m/s} \rightarrow \)
2. \( \vec{v}_{Cy} = 20 \sin 60^\circ = 17.32 \text{ m/s} \downarrow \)
3. \( \vec{v}_C = \vec{v}_B + \vec{v}_{Cz} \)
4. \( t = 17.32 = \sqrt{(v_{Cy})^2 + (v_{Cz})^2} \)
5. \( v_{Cy} = \sqrt{(4.25)^2 + (17.32)^2} = 17.8 \text{ m/s} \quad \text{Ans} \)
6. \( \theta = \tan^{-1} \frac{17.32}{4.25} = 76.2^\circ \quad \text{Ans} \)
7. \( \vec{a}_C = \vec{a}_B + \vec{a}_{Cz} \)
8. \( -9.81 \hat{f} = \vec{a}_B + \vec{a}_{Cz} \)
9. \( \vec{a}_{Cz} = 9.81 \text{ m/s}^2 \hat{f} \quad \text{Ans} \)