Solution of Homework Problems
(Chapter 13)

Summer 2008
13-2. By using an inclined plane to retard the motion of a falling object, and thus make the observations more accurate, Galileo was able to determine experimentally that the distance through which an object moves in free fall is proportional to the square of the time for travel. Show that this is the case, i.e., \( s \propto t^2 \), by determining the time \( t_A \), \( t_C \), and \( t_D \) needed for a block of mass \( m \) to slide from rest at \( A \) to points \( B \), \( C \), and \( D \), respectively. Neglect the effects of friction.

\[
\begin{align*}
\text{W sin} 20^\circ &= \frac{W}{g} \\
\text{\textit{a}} &= 9.81(\sin 20^\circ) = 3.355 \text{ m/s}^2 \\
\text{s} &= \frac{1}{2} \text{at}^2
\end{align*}
\]

<table>
<thead>
<tr>
<th>( s )</th>
<th>( t )</th>
<th>Ans</th>
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<tbody>
<tr>
<td>2 m</td>
<td>1.09 s</td>
<td>Ans</td>
</tr>
<tr>
<td>4 m</td>
<td>1.54 s</td>
<td>Ans</td>
</tr>
<tr>
<td>9 m</td>
<td>2.32 s</td>
<td>Ans</td>
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13-9. The elevator \( E \) has a mass of 500 kg, and the counterweight at \( A \) has a mass of 150 kg. If the motor supplies a constant force of 5 kN on the cable at \( B \), determine the speed of the elevator when \( t = 3 \) s, starting from rest. Neglect the mass of the pulleys and cable.

For \( A \):
\[
\begin{align*}
+ \Sigma F_x &= ma_x: & 150(9.81) - T &= 150a_x & (1)
\end{align*}
\]

For \( E \):
\[
\begin{align*}
+ \Sigma F_x &= ma_x: & 500(9.81) - 5000 - T &= 500a_x & (2)
\end{align*}
\]

\( t_A + t_E = l \)

\( a_E = -a_A \) \( (3) \)

Solving:

\( T = 1110 \) N

\( a_E = -2.410 \text{ m/s}^2 = 2.410 \text{ m/s}^2 \) \( \uparrow \)

\( v = v_0 + a_x (t) \)

\( v_E = 0 + 2.410(3) = 7.23 \text{ m/s} \) \( \uparrow \) \( \text{Ans} \)
13-14. Each of the two blocks has a mass $m$. The coefficient of kinetic friction at all surfaces of contact is $\mu$. If a horizontal force $P$ moves the bottom block, determine the acceleration of the bottom block in each case.

(a) Block $A$:

$$\Sigma F_i = ma_i; \quad P - 3\mu mg = ma_A$$

$$a_i = \frac{P}{m} - 3\mu g \quad \text{Ans}$$

(b) $s_B + s_A = l$

$$a_A = -a_B \quad (1)$$

Block $A$:

$$\Sigma F_i = ma_i; \quad P - T - 3\mu mg = ma_A \quad (2)$$

Block $B$:

$$\Sigma F_i = ma_i; \quad \mu mg - T = ma_B \quad (3)$$

Subtract Eq. (3) from Eq. (2):

$$P - 4\mu mg = m(a_A - a_B)$$

Use Eq. (1):

$$a_A = \frac{P}{2m} - 2\mu g \quad \text{Ans}$$

19. A 40-lb suitcase slides from rest 20 ft down the 30° ramp. Determine the point where it strikes the ground at $C$. How long does it take to go from $A$ to $C$?

$$\Sigma F_i = ma_i; \quad 40 \sin 30° = \frac{40}{32.2} a$$

$$a = 16.1 \text{ ft/s}^2$$

$$v^2 = v_0^2 + 2a(s - s_0); \quad v = 0 + 2(16.1)(20)$$

$$v_B = 25.38 \text{ ft/s}$$

$$v = v_0 + a_1 T; \quad 25.38 = 0 + 16.1 t_{AB}$$

$$t_{AB} = 1.576 \text{ s}$$

$$s = (s_f)_0 + (v_f)_0 t$$

$$R = 0 + 25.38 \cos 30° (t_{BC})$$

$$\frac{s}{4} = (s_f)_0 + (v_f)_0 t + \frac{1}{2}a_1 t^2$$

$$4 = 0 + 25.38 \sin 30° t_{BC} + \frac{1}{2}(32.2)(t_{BC})^2$$

$$t_{BC} = 0.2413 \text{ s}$$

$$R = 5.30 \text{ ft}$$

Total time $= t_{AB} + t_{BC} = 1.82 \text{ s} \quad \text{Ans}$
13-42. Blocks A and B each have a mass $m$. Determine the largest horizontal force $P$ which can be applied to $B$ so that $A$ will not move relative to $B$. All surfaces are smooth.

Require

\[ a_A = a_B = a \]
Block A:

\[ \sum F_x = 0; \quad N \cos \theta - mg = 0 \]
\[ \sum F_y = ma; \quad N \sin \theta = ma \]
\[ a = g \tan \theta \]

Block B:

\[ \sum F_x = ma; \quad P - N \sin \theta = ma \]
\[ P - mg \tan \theta = mg \tan \theta \]
\[ P = 2mg \tan \theta \quad \text{Ans} \]

13-57. The 600-kg wrecking ball is suspended from the crane by a cable having a negligible mass. If the ball has a speed $v = 8$ m/s at the instant it is at its lowest point, $\theta = 0^\circ$, determine the tension in the cable at this instant. Also, determine the angle $\theta$ to which the ball swings before it stops.

\[ + \sum F_y = ma; \quad T - 600(9.81) = 600(20^\circ) \]
\[ T = 9086 \text{ N} = 9.09 \text{ kN} \quad \text{Ans} \]

\[ -9.81(12) \int_0^\theta \sin \theta \, d\theta - \int_0^\theta \nu \, dv \]
\[ -9.81(12)(-\cos \theta + 1) = \frac{1}{2}v^2 \]
\[ \theta = 43.3^\circ \quad \text{Ans} \]

13-65. The 150-lb man lies against the cushion for which the coefficient of static friction is $\mu_s = 0.5$. Determine the resultant normal and frictional forces the cushion exerts on him if, due to rotation about the $z$ axis, he has a constant speed $v = 20$ ft/s. Neglect the size of the man. Take $\theta = 60^\circ$.

\[ + \sum F_y = m(a_y); \quad N - 150 \cos 60^\circ = \frac{150}{32.2} \left( \frac{20^2}{8} \right) \sin 60^\circ \]
\[ N = 277 \text{ lb} \quad \text{Ans} \]

\[ + \sum F_x = m(a_x); \quad -F + 150 \sin 60^\circ = \frac{150}{32.2} \left( \frac{20^2}{8} \right) \cos 60^\circ \]
\[ F = 13.4 \text{ lb} \quad \text{Ans} \]

Note: No slipping occurs

Since $\mu_s N = 138.4 \text{ lb} > 13.4 \text{ lb}$
13-77. The 35-kg box has a speed of 2 m/s when it is at A on the smooth ramp. If the surface is in the shape of a parabola, determine the normal force on the box at the instant \( x = 3 \) m. Also, what is the rate of increase in its speed at this instant?

\[
y = 4 - \frac{1}{9}x^2
\]

\[
\frac{dy}{dx} = \tan \theta = -\frac{1}{\frac{1}{9}} \bigg|_{x=3} = -0.6667 \quad \theta = -33.69^\circ
\]

\[
\frac{d^2y}{dx^2} = \frac{2}{9}
\]

\[
\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + \left(-\frac{2x}{9}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{2}{9}\right|} = 4.5 \left(1 + 0.04938x^2\right)^{\frac{3}{2}} \bigg|_{x=3} = 7.812 \text{ m}
\]

\[
\Sigma F_x = ma_x: \quad 35(9.81) \cos \theta - N = 35 \left(\frac{v^2}{7.812}\right) \quad (1)
\]

\[
\Sigma F_y = ma_y: \quad 35(9.81) \sin \theta = 35a_y
\]

\[a_y = 9.81 \sin \theta \quad (2)
\]

\[
\nu \, dv = a_x \, ds
\]

\[
\nu \, dv = 9.81 \sin \theta \, ds
\]

\[
\nu \, dv = 9.81 \left(\frac{dy}{dx}\right) \, ds = -9.81 \, dy
\]

When \( x = 0, \ y = 4 \). When \( x = 3, \ y = 4 - \frac{1}{9}(3)^2 = 3 \). Thus,

\[
\int_{x_1}^{x_2} \nu \, dv = -\int_4^3 9.81 \, dy
\]

\[
\frac{1}{2} v^2 - \frac{1}{2} (2)^2 = -9.81(3 - 4)
\]

\[
v = 4.86 \text{ m/s}
\]

From Eqs. (1) and (2) for \( \theta = 33.69^\circ \)

\[35(9.81) \cos 33.69^\circ - N = 35 \left(\frac{4.86^2}{7.812}\right) \]

\[N = 180 \text{ N} \quad \text{Ans}
\]

\[a_y = 9.81 \sin 33.69^\circ = 5.44 \text{ m/s}^2 \quad \text{Ans}\]
13-92. Solve Problem 13-91 if the arm has an angular acceleration of $\ddot{\theta} = 3 \text{ rad/s}^2$ and $\dot{\theta} = 2 \text{ rad/s}$ at this instant. Assume the particle contacts only one side of the slot at any instant.

$$r = \frac{0.5}{\cos \theta} = 0.5 \sec \theta$$

$$\dot{r} = 0.5 \sec \theta \tan \theta \dot{\theta}$$

$$\ddot{r} = 0.5 \left[ \left( \sec \theta \tan \theta \dot{\theta} + \sec \theta \left( \sec^2 \theta \ddot{\theta} \right) \right) \dot{\theta} + \sec \theta \tan \theta \dot{\theta} \ddot{\theta} \right]$$

$$= 0.5 \left[ \sec \theta \tan \theta \ddot{\theta} + \sec^3 \ddot{\theta} + \sec \theta \tan \theta \dot{\theta} \right]$$

When $\theta = 30^\circ$, $\dot{\theta} = 2 \text{ rad/s}$ and $\ddot{\theta} = 3 \text{ rad/s}^2$

$$r = 0.5 \sec 30^\circ = 0.5774 \text{ m}$$

$$\dot{r} = 0.5 \sec 30^\circ \tan 30^\circ (2) = 0.6667 \text{ m/s}$$

$$\ddot{r} = 0.5 \left[ \sec 30^\circ \tan^2 30^\circ (2)^2 + \sec^3 30^\circ (2)^2 + \sec 30^\circ \tan 30^\circ (3) \right]$$

$$= 4.849 \text{ m/s}^2$$

$$a = \ddot{r} - r \ddot{\theta} = 4.849 - 0.5774 (2)^2 = 2.5396 \text{ m/s}^2$$

$$a_n = r \ddot{\theta} + 2 \dddot{\theta} = 0.5774 (3) + 2 (0.6667)(2) = 4.3987 \text{ m/s}^2$$

$$\sum F_x = m a_x; \quad N \cos 30^\circ - 0.5 (9.81) \cos 30^\circ = 0.5 (2.5396)$$

$$N = 6.3712 = 6.37 \text{ N} \quad \text{Ans}$$

$$\sum F_y = m a_y; \quad F + 0.5 (9.81) \sin 30^\circ - 6.3712 \sin 30^\circ = 0.5 (4.3987)$$

$$F = 2.93 \text{ N} \quad \text{Ans}$$
The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limacon, \( r = (2 + \cos \theta) \) ft. If \( \theta = (0.5 \pi^2) \) rad, where \( t \) is in seconds, determine the force which the rod exerts on the particle at the instant \( t = 1 \) s. The fork and path contact the particle on only one side.

\[
\begin{align*}
\rho &= 2 - \cos \theta \\
\rho &= 0.5 \pi^2 \\
\rho &= -\sin \theta \\
\rho &= 1 \text{ rad/s}^2
\end{align*}
\]

At \( t = 1 \) s, \( \theta = 0.5 \) rad, \( \dot{\theta} = 1 \) rad/s and \( \ddot{\theta} = 1 \) rad/s^2

\[
\begin{align*}
r &= 2 + \cos 0.5 = 2.8776 \text{ ft} \\
\dot{r} &= -\sin 0.5(1) = -0.4794 \text{ ft/s} \\
\ddot{r} &= -\cos 0.5(1) - \sin 0.5(1) = -1.357 \text{ ft/s}^2 \\
a_x &= \ddot{r} - r\dddot{\theta} = -1.357 - 2.8776(1)^2 = -4.2346 \text{ ft/s}^2 \\
a_y &= r\dddot{\theta} + 2\ddot{r} = 2.8776(1) + 2(-0.4794)(1) = 1.9187 \text{ ft/s}^2
\end{align*}
\]

\[
\begin{align*}
\tan \psi &= \frac{r}{\dot{r}} = \frac{2 + \cos \theta}{-\sin \theta} & \theta = 0.5 \pi \text{ rad} & \psi = -80.34^\circ
\end{align*}
\]

\[
\begin{align*}
\Sigma F_x &= m\ddot{x}; \\
-0.2666 \sin 90^\circ &= \frac{2}{32.2}(-4.2346) & N = 0.2666 \text{ lb}
\end{align*}
\]

\[
\begin{align*}
\Sigma F_y &= m\ddot{y}; \\
-0.2666 \cos 90^\circ &= \frac{2}{32.2}(1.9187) & F = 0.164 \text{ lb}
\end{align*}
\]
The ball has a mass of 2 kg and a negligible size. It is originally traveling around the horizontal circular path of radius $r_0 = 0.5$ m such that the angular rate of rotation is $\dot{\theta} = 1$ rad/s. If the attached cord $ABC$ is drawn down through the hole at a constant speed of 0.2 m/s, determine the tension the cord exerts on the ball at the instant $r = 0.25$ m. Also, compute the angular velocity of the ball at this instant. Neglect the effects of friction between the ball and horizontal plane. 

**Hint:** First show that the equation of motion in the $\theta$ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)(d(r^2\dot{\theta})/dt) = 0$. When integrated, $r^2\dot{\theta} = c$, where the constant $c$ is determined from the problem data.

\[
\sum F_\theta = ma_\theta: \quad 0 = m[r\ddot{\theta} + 2\dot{r}\dot{\theta}] = m \left[ \frac{1}{r} \frac{d}{dt}(r^2\dot{\theta}) \right] = 0
\]

Thus,

\[
d(r^2\dot{\theta}) = 0
\]

\[
r^2\dot{\theta} = c
\]

\[
0.5^2(1) = c = (0.25)^2\dot{\theta}
\]

\[
\dot{\theta} = 4.00 \text{ rad/s} \quad \text{Ans}
\]

Since $\dot{r} = 0.2$ m/s. $\ddot{r} = 0$

\[
a_r = \dot{r} - r(\dot{\theta})^2 = 0 - 0.25(4.00)^2 = -4 \text{ m/s}^2
\]

\[
\sum a_r = ma_r: \quad -T = 2(-4)
\]

\[
T = 8 \text{ N} \quad \text{Ans}
\]