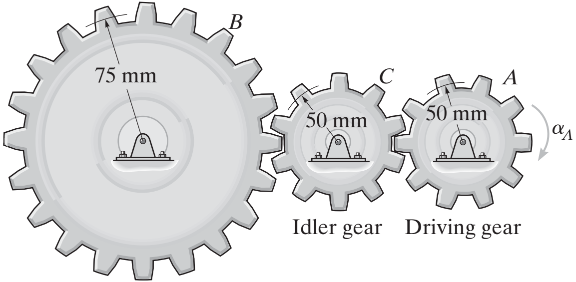
•**16–9.** When only two gears are in mesh, the driving gear $A$ and the driven gear $B$ will always turn in opposite directions. In order to get them to turn in the *same direction* an idler gear $C$ is used. In the case shown, determine the angular velocity of gear $B$ when $t = 5$ s, if gear $A$ starts from rest and has an angular acceleration of $\alpha_A = (3t + 2)$ rad/s$^2$, where $t$ is in seconds.




$$d\omega = \alpha\, dt$$

$$\int_0^{\omega_A} d\omega_A = \int_0^t (3t + 2)\, dt$$

$$\omega_A = 1.5t^2 + 2t\big|_{t=5} = 47.5 \text{ rad/s}$$

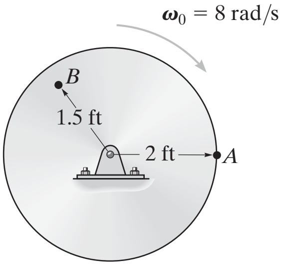
$$(47.5)(50) = \omega_C\,(50)$$

$$\omega_C = 47.5 \text{ rad/s}$$

$$\omega_B\,(75) = 47.5(50)$$

$$\omega_B = 31.7 \text{ rad/s}$$ **Ans.**

**16–22.** The disk is originally rotating at $\omega_0 = 8$ rad/s. If it is subjected to a constant angular acceleration of $\alpha = 6$ rad/s$^2$, determine the magnitudes of the velocity and the $n$ and $t$ components of acceleration of point $B$ just after the wheel undergoes 2 revolutions.




$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega^2 = (8)^2 + 2(6)[2(2\pi) - 0]$$

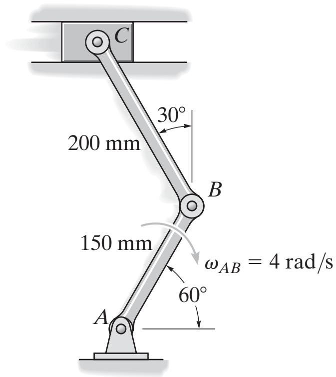
$$\omega = 14.66 \text{ rad/s}$$

$$v_B = \omega r = 14.66(1.5) = 22.0 \text{ ft/s}$$ **Ans.**

$$(a_B)_t = \alpha r = 6(1.5) = 9.00 \text{ ft/s}^2$$ **Ans.**

$$(a_B)_n = \omega^2 r = (14.66)^2(1.5) = 322 \text{ ft/s}^2$$ **Ans.**

***16–68.** If bar $AB$ has an angular velocity $\omega_{AB} = 4$ rad/s, determine the velocity of the slider block $C$ at the instant shown.




For link $AB$: Link $AB$ rotates about a fixed point $A$. Hence

$$v_B = \omega_{AB}\, r_{AB} = 4(0.15) = 0.6 \text{ m/s}$$

For link $BC$

$$\mathbf{v}_B = \{0.6\cos 30°\mathbf{i} - 0.6\sin 30°\mathbf{j}\}\text{m/s} \qquad \mathbf{v}_C = v_C\mathbf{i} \qquad \boldsymbol{\omega} = \omega_{BC}\,\mathbf{k}$$

$$\mathbf{r}_{C/B} = \{-0.2\sin 30°\mathbf{i} + 0.2\cos 30°\mathbf{j}\}\text{ m}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$$

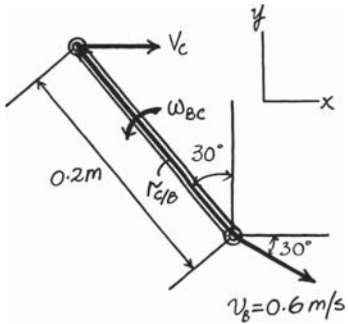
$$v_C\,\mathbf{i} = (0.6\cos 30°\mathbf{i} - 0.6\sin 30°\mathbf{j}) + (\omega_{BC}\,\mathbf{k}) \times (-0.2\sin 30°\mathbf{i} + 0.2\cos 30°\mathbf{j})$$

$$v_C\mathbf{i} = (0.5196 - 0.1732\omega_{BC})\mathbf{i} - (0.3 + 0.1\omega_{BC})\mathbf{j}$$

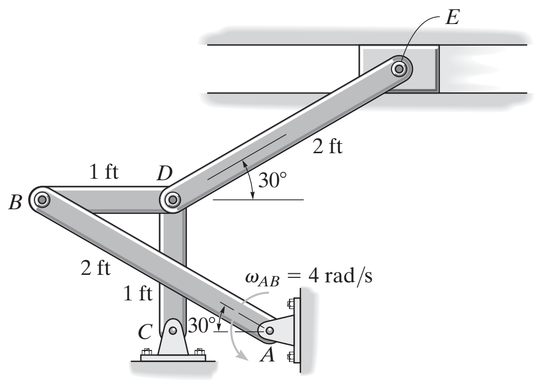
Equating the **i** and **j** components yields:

$$0 = 0.3 + 0.1\omega_{BC} \qquad \omega_{BC} = -3 \text{ rad/s}$$

$$v_C = 0.5196 - 0.1732(-3) = 1.04 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$



•**16–73.** If link $AB$ has an angular velocity of $\omega_{AB} = 4$ rad/s at the instant shown, determine the velocity of the slider block $E$ at this instant. Also, identify the type of motion of each of the four links.

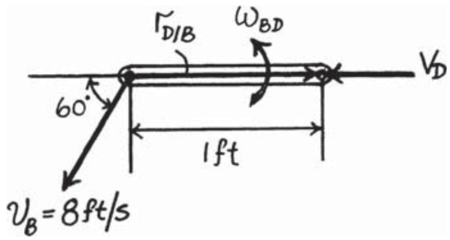



Link $AB$ rotates about the fixed point $A$. Hence

$$v_B = \omega_{AB}\, r_{AB} = 4(2) = 8 \text{ ft/s}$$

For link $BD$

$$\mathbf{v}_B = \{-8\cos 60^\circ\mathbf{i} - 8\sin 60^\circ\,\mathbf{j}\}\text{ ft/s} \qquad \mathbf{v}_D = -v_D\mathbf{i} \qquad \boldsymbol{\omega}_{BD} = \omega_{BD}\,\mathbf{k}$$




$$\mathbf{r}_{D/B} = \{1\mathbf{i}\}\text{ ft}$$

$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B}$$

$$-v_D\mathbf{i} = (-8\cos 60^\circ\mathbf{i} - 8\sin 60^\circ\mathbf{j}) + (\omega_{BD}\mathbf{k}) \times (1\mathbf{i})$$

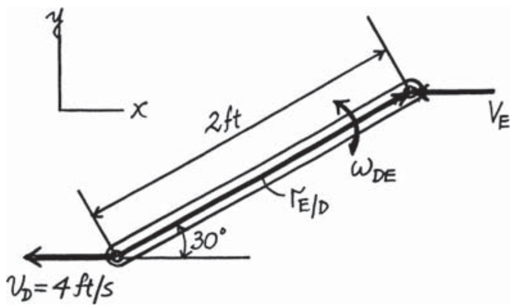
$$-v_D\,\mathbf{i} = -8\cos 60^\circ\mathbf{i} + (\omega_{BD} - 8\sin 60^\circ)\mathbf{j}$$

$(\overset{+}{\rightarrow})$ $\quad -v_D = -8\cos 60^\circ \qquad v_D = 4 \text{ ft/s}$

$(+\uparrow)$ $\quad 0 = \omega_{BD} - 8\sin 60^\circ \qquad \omega_{BD} = 6.928 \text{ rad/s}$

For Link $DE$

$$\mathbf{v}_D = \{-4\mathbf{i}\}\text{ ft/s} \qquad \boldsymbol{\omega}_{DE} = \omega_{DE}\,\mathbf{k} \qquad \mathbf{v}_E = -v_E\mathbf{i}$$




$$\mathbf{r}_{E/D} = \{2\cos 30^\circ\mathbf{i} + 2\sin 30^\circ\mathbf{j}\}\text{ ft}$$

$$\mathbf{v}_E = \mathbf{v}_D + \boldsymbol{\omega}_{DE} \times \mathbf{r}_{E/D}$$

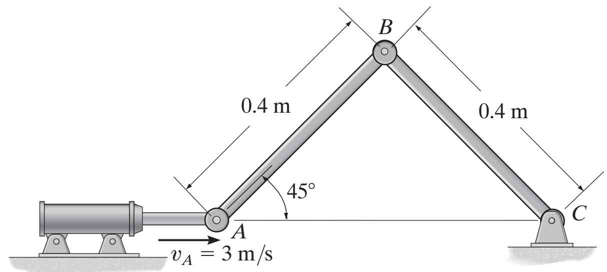
$$-v_E\mathbf{i} = -4\mathbf{i} + (\omega_{DE}\mathbf{k}) \times (2\cos 30^\circ\mathbf{i} + 2\sin 30^\circ\mathbf{j})$$

$$-v_E\mathbf{i} = (-4 - 2\sin 30^\circ\,\omega_{DE})\mathbf{i} + 2\cos 30^\circ\omega_{DE}\mathbf{j}$$

$(\overset{+}{\rightarrow})$ $\quad 0 = 2\cos 30^\circ\,\omega_{DE} \qquad \omega_{DE} = 0$

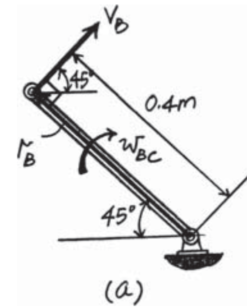
$(+\uparrow)$ $\quad -v_E = -4 - 2\sin 30^\circ(0) \qquad v_E = 4\text{ft/s} \quad \leftarrow \qquad$ **Ans.**

•**16–93.** If end $A$ of the hydraulic cylinder is moving with a velocity of $v_A = 3\ \text{m/s}$, determine the angular velocity of rod $BC$ at the instant shown.




**Rotation About a Fixed Axis:** Referring to Fig. *a*,

$$v_B = \omega_{BC}\, r_B = \omega_{BC}\,(0.4)$$



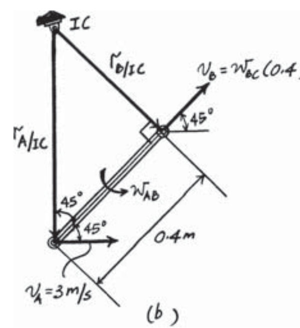

**General Plane Motion:** The location of the $IC$ for rod $AB$ is indicated in Fig. *b*. From the geometry shown in this figure, we obtain

$$r_{A/IC} = \frac{0.4}{\cos 45°} \qquad r_{A/IC} = 0.5657\ \text{m}$$

$$r_{B/IC} = 0.4 \tan 45° = 0.4\ \text{m}$$

Thus, the angular velocity of rod $AB$ can be determined from

$$\omega_{AB} = \frac{v_A}{r_{A/IC}} = \frac{3}{0.5657} = 5.303\ \text{rad/s}$$



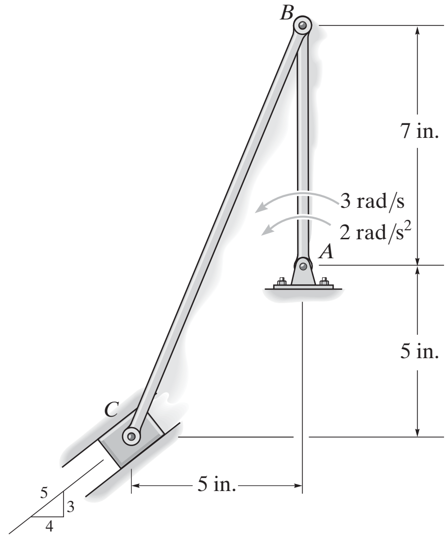

Then,

$$v_B = \omega_{AB}\, r_{B/IC}$$

$$\omega_{BC}\,(0.4) = 5.303(0.4)$$

$$\omega_{BC} = 5.30\ \text{rad/s} \qquad \textbf{Ans.}$$

***16–116.** At the given instant member $AB$ has the angular motions shown. Determine the velocity and acceleration of the slider block $C$ at this instant.



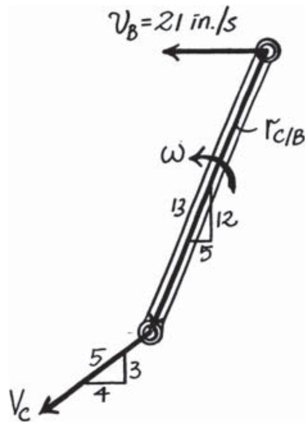

$v_B = 3(7) = 21 \text{ in./s} \leftarrow$

$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$

$$-v_C\left(\frac{4}{5}\right)\mathbf{i} - v_C\left(\frac{3}{5}\right)\mathbf{j} = -21\mathbf{i} + \omega\mathbf{k} \times (-5\mathbf{i} - 12\mathbf{j})$$

$(\overset{+}{\rightarrow}) \qquad -0.8v_C = -21 + 12\omega$

$(+\uparrow) \qquad -0.6v_C = -5\omega$




Solving:

$\omega = 1.125 \text{ rad/s}$

$v_C = 9.375 \text{ in./s} = 9.38 \text{ in./s}$ **Ans.**

$(a_B)_n = (3)^2(7) = 63 \text{ in./s}^2 \downarrow$

$(a_B)_t = (2)(7) = 14 \text{ in./s}^2 \leftarrow$

$\mathbf{a}_C = a_B + \alpha \times \mathbf{r}_{C/B} - \omega^2 \mathbf{r}_{C/B}$

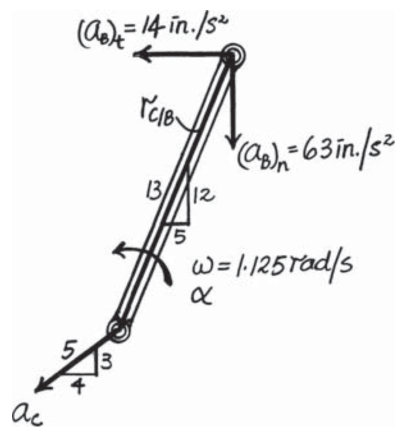
$$-a_C\left(\frac{4}{5}\right)\mathbf{i} - a_C\left(\frac{3}{5}\right)\mathbf{j} = -14\mathbf{i} - 63\mathbf{j} + (\alpha\mathbf{k}) \times (-5\mathbf{i} - 12\mathbf{j}) - (1.125)^2(-5\mathbf{i} - 12\mathbf{j})$$

$(\overset{+}{\rightarrow}) \qquad -0.8a_C = -14 + 12\alpha + 6.328$

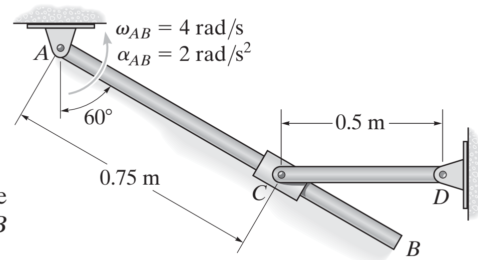
$(+\uparrow) \qquad -0.6a_C = -63 - 5\alpha + 15.1875$

$a_C = 54.7 \text{ in./s}^2$ **Ans.**

$\alpha = -3.00 \text{ rad/s}^2$



**16–142.** At the instant shown rod $AB$ has an angular velocity $\omega_{AB} = 4$ rad/s and an angular acceleration $\alpha_{AB} = 2$ rad/s$^2$. Determine the angular velocity and angular acceleration of rod $CD$ at this instant. The collar at $C$ is pin connected to $CD$ and slides freely along $AB$.




**Coordinate Axes:** The origin of both the fixed and moving frames of reference are located at point $A$. The $x$, $y$, $z$ moving frame is attached to and rotate with rod $AB$ since collar $C$ slides along rod $AB$.

**Kinematic Equation:** Applying Eqs. 16–24 and 16–27, we have

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz} \qquad [1]$$

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (\mathbf{v}_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz} \qquad [2]$$

*Motion of moving reference*

$\mathbf{v}_A = \mathbf{0}$

$\mathbf{a}_A = \mathbf{0}$

$\Omega = 4\mathbf{k}$ rad/s

$\dot{\Omega} = 2\mathbf{k}$ rad/s$^2$

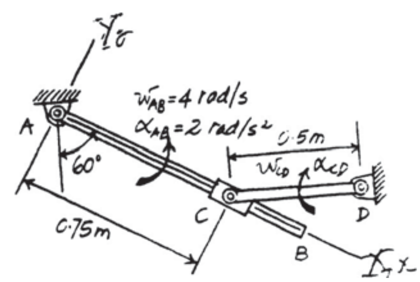
*Motion of C with respect to moving reference*

$r_{C/A} = \{0.75\mathbf{i}\}$m

$(\mathbf{v}_{C/A})_{xyz} = (v_{C/A})_{xyz}\,\mathbf{i}$

$(\mathbf{a}_{C/A})_{xyz} = (a_{C/A})_{xyz}\,\mathbf{i}$

The velocity and acceleration of collar $C$ can be determined using Eqs. 16–9 and 16–14 with $\mathbf{r}_{C/D} = \{-0.5\cos 30°\mathbf{i} - 0.5\sin 30°\mathbf{j}\}$m $= \{-0.4330\mathbf{i} - 0.250\mathbf{j}\}$ m.




$$\mathbf{v}_C = \omega_{CD} \times \mathbf{r}_{C/D} = -\omega_{CD}\mathbf{k} \times (-0.4330\mathbf{i} - 0.250\mathbf{j})$$

$$= -0.250\omega_{CD}\mathbf{i} + 0.4330\omega_{CD}\mathbf{j}$$

$$\mathbf{a}_C = \alpha_{CD} \times \mathbf{r}_{C/D} - \omega_{CD}^2\mathbf{r}_{C/D}$$

$$= -\alpha_{CD}\,\mathbf{k} \times (-0.4330\mathbf{i} - 0.250\mathbf{j}) - \omega_{CD}^2(-0.4330\mathbf{i} - 0.250\mathbf{j})$$

$$= \left(0.4330\omega_{CD}^2 - 0.250\,\alpha_{CD}\right)\mathbf{i} + \left(0.4330\alpha_{CD} + 0.250\omega_{CD}^2\right)\mathbf{j}$$

Substitute the above data into Eq.[1] yields

$$\mathbf{v}_C = \mathbf{v}_A + \Omega \times \mathbf{r}_{C/A} + (\mathbf{v}_{C/A})_{xyz}$$

$$-0.250\,\omega_{CD}\,\mathbf{i} + 0.4330\omega_{CD}\mathbf{j} = \mathbf{0} + 4\mathbf{k} \times 0.75\mathbf{i} + (v_{C/A})_{xyz}\,\mathbf{i}$$

$$-0.250\omega_{CD}\mathbf{i} + 0.4330\omega_{CD}\mathbf{j} = (v_{C/A})_{xyz}\,\mathbf{i} + 3.00\mathbf{j}$$

Equating **i** and **j** components and solve, we have

$$(v_{C/A})_{xyz} = -1.732 \text{ m/s}$$

$$\omega_{CD} = 6.928 \text{ rad/s} = 6.93 \text{ rad/s} \qquad \textbf{Ans.}$$

Substitute the above data into Eq.[2] yields

$$\mathbf{a}_C = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{C/A} + \Omega \times (\Omega \times \mathbf{r}_{C/A}) + 2\Omega \times (v_{C/A})_{xyz} + (\mathbf{a}_{C/A})_{xyz}$$

$$\left[0.4330\left(6.928^2\right) - 0.250\,\alpha_{CD}\right]\mathbf{i} + \left[0.4330\alpha_{CD} + 0.250\left(6.928^2\right)\right]\mathbf{j}$$

$$= \mathbf{0} + 2\mathbf{k} \times 0.75\mathbf{i} + 4\mathbf{k} \times (4\mathbf{k} \times 0.75\mathbf{i}) + \mathbf{2}\,(4\mathbf{k}) \times (-1.732\mathbf{i}) + (a_{C/A})_{xyz}\,\mathbf{i}$$

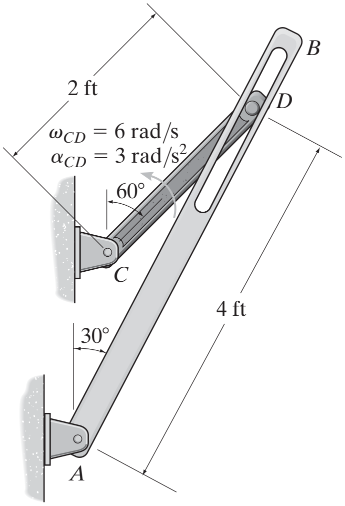
$$(20.78 - 0.250\alpha_{CD})\mathbf{i} + (0.4330\,\alpha_{CD} + 12)\mathbf{j} = \left[(a_{C/A})_{xyz} - 12.0\right]\mathbf{i} - 12.36\mathbf{j}$$

Equating **i** and **j** components, we have

$$(a_{C/A})_{xyz} = 46.85 \text{ m/s}^2$$

$$\alpha_{CD} = -56.2 \text{ rad/s}^2 = 56.2 \text{ rad/s}^2 \circlearrowright \qquad \textbf{Ans.}$$

**16–159.** The quick return mechanism consists of the crank $CD$ and the slotted arm $AB$. If the crank rotates with the angular velocity and angular acceleration at the instant shown, determine the angular velocity and angular acceleration of $AB$ at this instant.



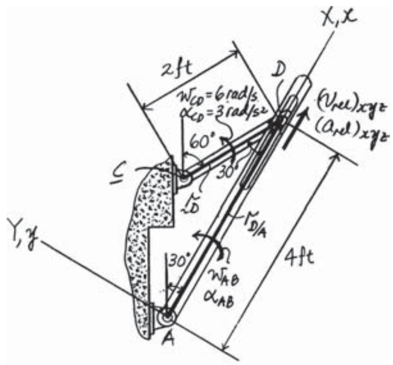

**Reference Frame**: The $xyz$ rotating reference frame is attached to slotted arm $AB$ and coincides with the $XYZ$ fixed reference frame at the instant considered, Fig. $a$. Thus, the motion of the $xyz$ reference frame with respect to the $XYZ$ frame is

$$\mathbf{v}_A = a_A = \mathbf{0} \qquad \omega_{AB} = \omega_{AB}\mathbf{k} \qquad \dot{\omega}_{AB} = \alpha_{AB}\,\mathbf{k}$$

For the motion of point $D$ with respect to the $xyz$ frame, we have

$$\mathbf{r}_{D/A} = [4\mathbf{i}] \text{ ft} \qquad (\mathbf{v}_{\text{rel}})_{xyz} = (v_{\text{rel}})_{xyz}\mathbf{i} \qquad (\mathbf{a}_{\text{rel}})_{xyz} = (a_{\text{rel}})_{xyz}\,\mathbf{i}$$

Since the crank $CD$ rotates about a fixed axis, $\mathbf{v}_D$ and $\mathbf{a}_D$ with respect to the $XYZ$ reference frame can be determined from




(a)

$$\mathbf{v}_D = \omega_{CD} \times \mathbf{r}_D$$

$$= (6\mathbf{k}) \times (2\cos 30°\,\mathbf{i} - 2\sin 30°\,\mathbf{j})$$

$$= [6\mathbf{i} + 10.39\mathbf{j}] \text{ ft/s}$$

$$\mathbf{a}_D = \alpha_{CD} \times \mathbf{r}_D - \omega_{CD}{}^2\mathbf{r}_D$$

$$= (3\mathbf{k}) \times (2\cos 30°\,\mathbf{i} - 2\sin 30°\,\mathbf{j}) - 6^2(2\cos 30°\,\mathbf{i} - 2\sin 30°\,\mathbf{j})$$

$$= [-59.35\mathbf{i} + 41.20\mathbf{j}] \text{ ft/s}^2$$

**Velocity:** Applying the relative velocity equation,

$$\mathbf{v}_D = \mathbf{v}_A + \omega_{AB} \times r_{D/A} + (\mathbf{v}_{\text{rel}})_{xyz}$$

$$6\mathbf{i} + 10.39\,\mathbf{j} = \mathbf{0} + (\omega_{AB}\mathbf{k}) \times (4\mathbf{i}) + (v_{\text{rel}})_{xyz}\,\mathbf{i}$$

$$6\mathbf{i} + 10.39\mathbf{j} = (v_{\text{rel}})_{xyz}\,\mathbf{i} + 4\omega_{AB}\,\mathbf{j}$$

Equating the **i** and **j** components yields

$$(v_{\text{rel}})_{xyz} = 6 \text{ ft/s}$$

$$10.39 = 4\omega_{AB} \qquad \omega_{AB} = 2.598 \text{ rad/s} = 2.60 \text{ rad/s} \quad \textbf{Ans.}$$

**Acceleration:** Applying the relative acceleration equation,

$$\mathbf{a}_D = \mathbf{a}_A + \dot{\omega}_{AB} \times \mathbf{r}_{D/A} + \omega_{AB} \times (\omega_{AB} \times \mathbf{r}_{AB}) + 2\omega_{AB} \times (\mathbf{v}_{\text{rel}})_{xyz} + (\mathbf{a}_{\text{rel}})_{xyz}$$

$$-59.35\mathbf{i} + 41.20\,\mathbf{j} = \mathbf{0} + (\alpha_{AB}\mathbf{k}) \times 4\mathbf{i} + 2.598\mathbf{k} \times [(2.598\mathbf{k}) \times (4\mathbf{i})] + 2(2.598\mathbf{k}) \times (6\mathbf{i}) + (\mathbf{a}_{\text{rel}})_{xyz}\,\mathbf{i}$$

$$-59.35\mathbf{i} + 41.20\,\mathbf{j} = \left[(a_{\text{rel}})_{xyz} - 27\right]\mathbf{i} + (4\alpha_{AB} + 31.18)\mathbf{j}$$

Equating the **i** and **j** components yields

$$41.20 = 4\alpha_{AB} + 31.18$$

$$\alpha_{AB} = 2.50 \text{ rad/s}^2 \qquad \textbf{Ans.}$$