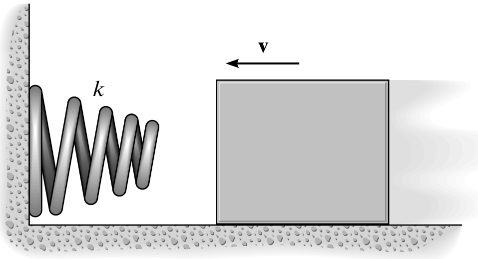
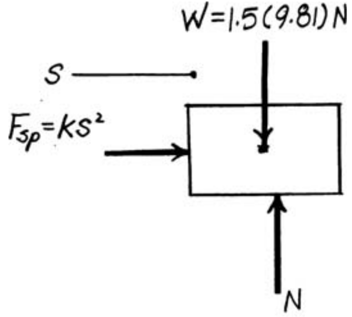
•**14–5.** The 1.5-kg block slides along a smooth plane and strikes a *nonlinear spring* with a speed of $v = 4$ m/s. The spring is termed "nonlinear" because it has a resistance of $F_s = ks^2$, where $k = 900\ \text{N/m}^2$. Determine the speed of the block after it has compressed the spring $s = 0.2$ m.



Principle of Work and Energy: The spring force F_{sp} which acts in the opposite direction to that of displacement does *negative* work. The normal reaction N and the weight of the block do not displace hence do no work. Applying Eq. 14–7, we have

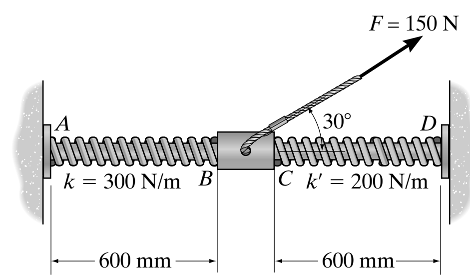


$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(1.5)\left(4^2\right) + \left[-\int_0^{0.2\text{ m}} 900 s^2\, ds\right] = \frac{1}{2}(1.5)\, v^2$$

$$v = 3.58\ \text{m/s} \qquad \textbf{Ans.}$$

•14–9. Springs AB and CD have a stiffness of $k = 300$ N/m and $k' = 200$ N/m, respectively, and both springs have an unstretched length of 600 mm. If the 2-kg smooth collar starts from rest when the springs are unstretched, determine the speed of the collar when it has moved 200 mm.

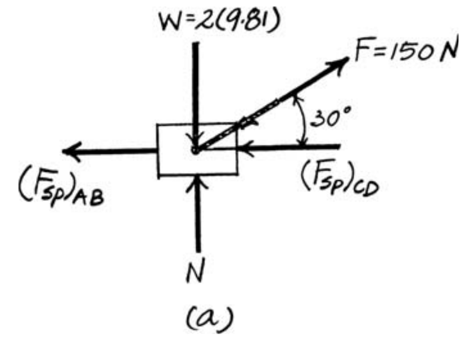


Principle of Work and Energy: By referring to the free-body diagram of the collar, notice that **W**, **N**, and $F_y = 150 \sin 30°$ do no work. However, $F_x = 150 \cos 30°$ N does positive work and $(\mathbf{F}_{sp})_{AB}$ and $(\mathbf{F}_{sp})_{CD}$ do negative work.

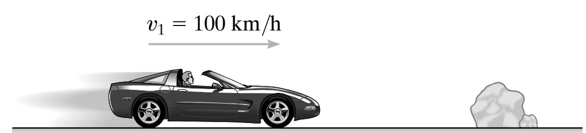
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 150 \cos 30°(0.2) + \left[-\frac{1}{2}(300)(0.2^2)\right] + \left[-\frac{1}{2}(200)(0.2^2)\right] = \frac{1}{2}(2)v^2$$

$$v = 4.00 \text{ m/s}$$ **Ans.**



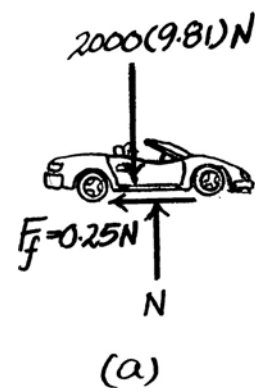
14–10. The 2-Mg car has a velocity of $v_1 = 100$ km/h when the driver sees an obstacle in front of the car. If it takes 0.75 s for him to react and lock the brakes, causing the car to skid, determine the distance the car travels before it stops. The coefficient of kinetic friction between the tires and the road is $\mu_k = 0.25$.



Free-Body Diagram: The normal reaction **N** on the car can be determined by writing the equation of motion along the y axis. By referring to the free-body diagram of the car, Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 2000(9.81) = 2000(0) \qquad N = 19\,620 \text{ N}$$

Since the car skids, the frictional force acting on the car is $F_f = \mu_k N = 0.25(19620) = 4905N$.



Principle of Work and Energy: By referring to Fig. a, notice that only $\mathbf{F}_f$ does work, which is negative. The initial speed of the car is $v_1 = \left[100(10^3)\frac{\text{m}}{\text{h}}\right]\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) =$ 27.78 m/s. Here, the skidding distance of the car is denoted as s'.

$$T_1 + \Sigma U_{1-2} = T_2$$

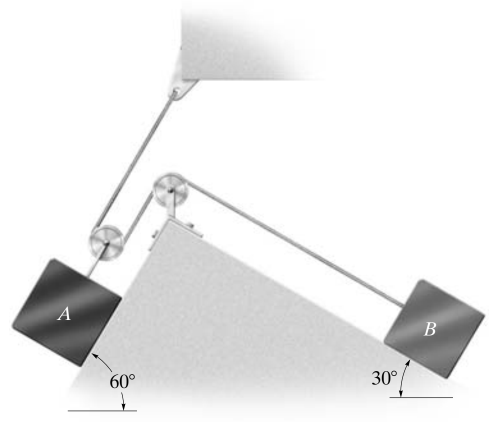
$$\frac{1}{2}(2000)(27.78^2) + (-4905s') = 0$$

$$s' = 157.31 \text{ m}$$

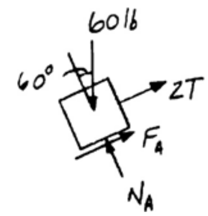
The distance traveled by the car during the reaction time is $s'' = v_1 t = 27.78(0.75) = 20.83$ m. Thus, the total distance traveled by the car before it stops is

$$s = s' + s'' = 157.31 + 20.83 = 178.14 \text{ m} = 178 \text{ m}$$ **Ans.**

14–13. Determine the velocity of the 60-lb block A if the two blocks are released from rest and the 40-lb block B moves 2 ft up the incline. The coefficient of kinetic friction between both blocks and the inclined planes is $\mu_k = 0.10$.



Block A:

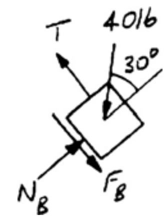


$+\nwarrow\Sigma F_y = ma_y$; $\quad N_A - 60\cos 60° = 0$

$N_A = 30$ lb

$F_A = 0.1(30) = 3$ lb

Block B:



$+\nearrow\Sigma F_y = ma_y$; $\quad N_B - 40\cos 30° = 0$

$N_B = 34.64$ lb

$F_B = 0.1(34.64) = 3.464$ lb

Use the system of both blocks. N_A, N_B, T, and R do no work.

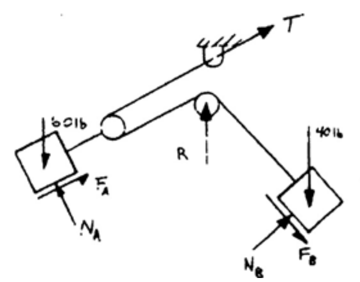
$T_1 + \Sigma U_{1-2} = T_2$

$$(0 + 0) + 60\sin 60°|\Delta s_A| - 40\sin 30°|\Delta s_B| - 3|\Delta s_A| - 3.464|\Delta s_B| = \frac{1}{2}\left(\frac{60}{32.2}\right)v_A^2 + \frac{1}{2}\left(\frac{40}{32.2}\right)v_B^2$$

$2s_A + s_B = l$

$2\Delta s_A = -\Delta s_B$

When $|\Delta s_B| = 2$ ft, $|\Delta s_A| = 1$ ft



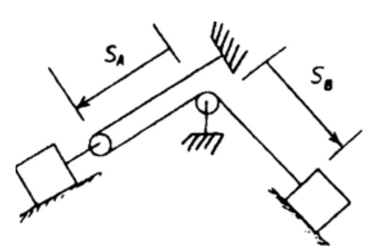
Also,

$2v_A = -v_B$

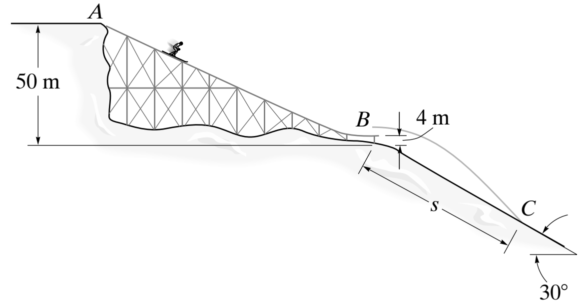
Substituting and solving,

$v_A = 0.771$ ft/s **Ans.**

$v_B = -1.54$ ft/s



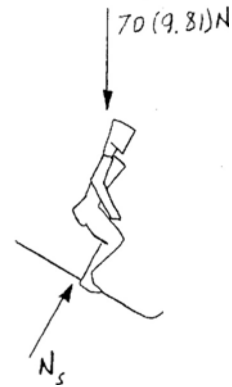
•**14–25.** The skier starts from rest at A and travels down the ramp. If friction and air resistance can be neglected, determine his speed v_B when he reaches B. Also, find the distance s to where he strikes the ground at C, if he makes the jump traveling horizontally at B. Neglect the skier's size. He has a mass of 70 kg.



$$T_A + \Sigma U_{A-B} = T_B$$

$$0 + 70(9.81)(46) = \frac{1}{2}(70)(v_B)^2$$

$$v_B = 30.04 \text{ m/s} = 30.0 \text{ m/s}$$ **Ans.**



$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$$s \cos 30° = 0 + 30.04t$$

$(+\downarrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$$s \sin 30° + 4 = 0 + 0 + \frac{1}{2}(9.81)t^2$$

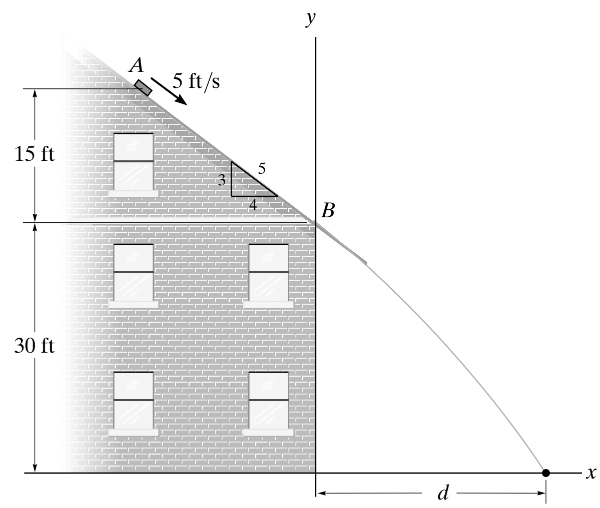
Eliminating t,

$$s^2 - 122.67s - 981.33 = 0$$

Solving for the positive root

$$s = 130 \text{ m}$$ **Ans.**

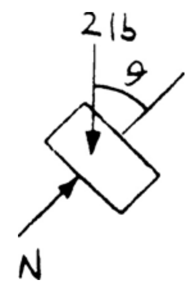
14–27. The 2-lb brick slides down a smooth roof, such that when it is at A it has a velocity of 5 ft/s. Determine the speed of the brick just before it leaves the surface at B, the distance d from the wall to where it strikes the ground, and the speed at which it hits the ground.



$$T_A + \Sigma U_{A-B} = T_B$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(15) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_B^2$$

$v_B = 31.48 \text{ ft/s} = 31.5 \text{ ft/s}$ **Ans.**



$\left(\overset{+}{\rightarrow}\right) \quad s = s_0 + v_0 t$

$$d = 0 + 31.48\left(\frac{4}{5}\right)t$$

$(+\downarrow) \quad s = s_0 + v_0 t - \frac{1}{2}a_c t^2$

$$30 = 0 + 31.48\left(\frac{3}{5}\right)t + \frac{1}{2}(32.2)t^2$$

$$16.1t^2 + 18.888t - 30 = 0$$

Solving for the positive root,

$t = 0.89916 \text{ s}$

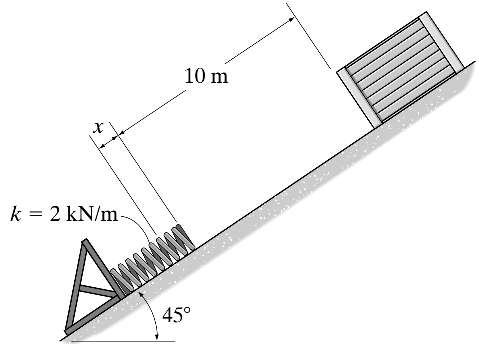
$d = 31.48\left(\frac{4}{5}\right)(0.89916) = 22.6 \text{ ft}$ **Ans.**

$$T_A + \Sigma U_{A-C} = T_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(5)^2 + 2(45) = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2$$

$v_C = 54.1 \text{ ft/s}$ **Ans.**

14–34. If the coefficient of kinetic friction between the 100-kg crate and the plane is $\mu_k = 0.25$, determine the speed of the crate at the instant the compression of the spring is $x = 1.5$ m. Initially the spring is unstretched and the crate is at rest.



Free-Body Diagram: The normal reaction **N** on the crate can be determined by writing the equation of motion along the y' axis and referring to the free-body diagram of the crate when it is in contact with the spring, Fig. *a*.

$$\nwarrow + F_{y'} = ma_{y'}; \qquad N - 100(9.81)\cos 45^\circ = 100(0) \qquad N = 693.67 \text{ N}$$

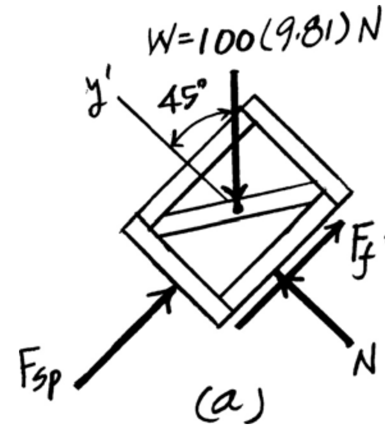
Thus, the frictional force acting on the crate is $F_f = \mu_k N = 0.25(693.67)$ N = 173.42 N. The force developed in the spring is $F_{sp} = kx = 2000x$.

Principle of Work and Energy: By referring to Fig. *a*, notice that **N** does no work. Here, **W** which displaces downward through a distance of $h = (10 + 1.5)\sin 45^\circ = 8.132$ m does positive work, whereas $\mathbf{F}_f$ and $\mathbf{F}_{sp}$ do negative work.

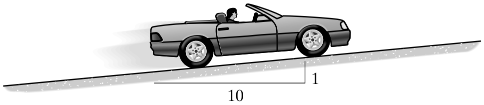
$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + 100(9.81)(8.132) + \left[-173.42(10 + 1.5)\right] + \left[-\frac{1}{2}(2000)(1.5^2)\right] = \frac{1}{2}(100)v^2$$

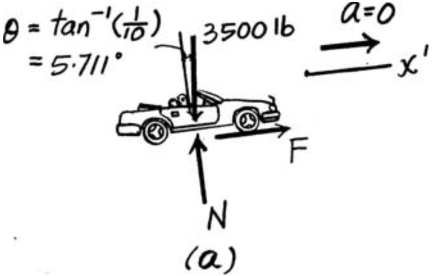
$$v = 8.64 \text{m/s}$$ **Ans.**



14–46. The engine of the 3500-lb car is generating a constant power of 50 hp while the car is traveling up the slope with a constant speed. If the engine is operating with an efficiency of $\epsilon = 0.8$, determine the speed of the car. Neglect drag and rolling resistance.



Equations of Motion: By referring to the free-body diagram of the car shown in Fig. *a*,



$$+\nearrow\Sigma F_{x'} = ma_{x'}; \qquad F - 3500 \sin 5.711° = \frac{3500}{32.2}(0) \qquad F = 348.26 \text{ lb}$$

Power: The power input of the car is $P_{in} = (50 \text{ hp})\left(\frac{550 \text{ ft} \cdot \text{lb/s}}{1 \text{ hp}}\right) = 27\,500 \text{ ft} \cdot \text{lb/s}$.

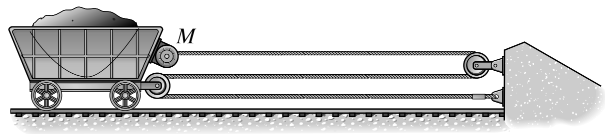
Thus, the power output is given by $P_{out} = \varepsilon P_{in} = 0.8(27\,500) = 22\,000 \text{ ft} \cdot \text{lb/s}$.

$$P_{out} = \mathbf{F} \cdot \mathbf{v}$$

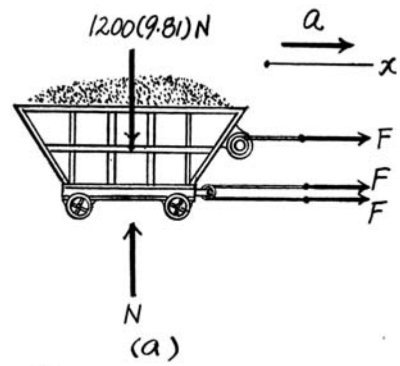
$$22\,000 = 348.26v$$

$$v = 63.2 \text{ ft/s}$$ **Ans.**

14–59. The 1.2-Mg mine car is being pulled by the winch M mounted on the car. If the winch generates a constant power output of 30 kW, determine the speed of the car at the instant it has traveled a distance of 30 m, starting from rest.



Equations of Motion: Here, $a = v\dfrac{dv}{ds}$. By referring to the free-body diagram of the mine car shown in Fig. a,



$\xrightarrow{+} \Sigma F_x = ma_x;$ $$3F = 1200\left(v\frac{dv}{ds}\right) \quad \textbf{(1)}$$

Power:

$$P_{\text{out}} = 3\mathbf{F}\cdot\mathbf{v}$$

$$30(10^3) = 3Fv \quad \textbf{(2)}$$

Substituting Eq. (1) into Eq. (2) yields

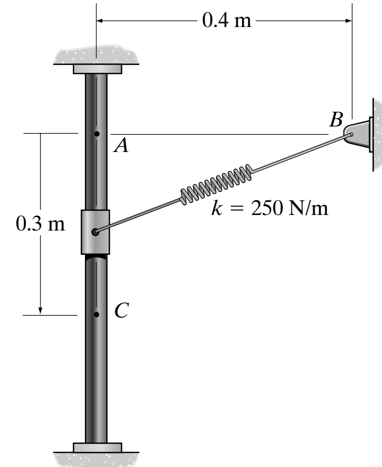
$$30(10^3) = 1200\left(v\frac{dv}{ds}\right)v$$

$$\int_0^v v^2 dv = \int_0^{30\text{ m}} 25 ds$$

$$\left.\frac{v^3}{3}\right|_0^v = 25s\Big|_0^{30\text{ m}}$$

$$v = 13.1 \text{ m/s} \qquad \textbf{Ans.}$$

14–83. The vertical guide is smooth and the 5-kg collar is released from rest at A. Determine the speed of the collar when it is at position C. The spring has an unstretched length of 300 mm.



Potential Energy: With reference to the datum set in Fig. a, the gravitational potential energy of the collar at positions A and C are $\left(V_g\right)_A = mgh_A = 5(9.81)(0) = 0$ and $\left(V_g\right)_C = mgh_C = 5(9.81)(-0.3) = -14.715$ J. When the collar is at positions A and C, the spring stretches $s_A = 0.4 - 0.3 = 0.1$ m and $s_C = \sqrt{0.4^2 + 0.3^2} - 0.3 = 0.2$ m. The elastic potential energy of the spring when the collar is at these two positions are $\left(V_e\right)_A = \frac{1}{2}ks_A{}^2 = \frac{1}{2}(250)(0.1^2) = 1.25$ J and $\left(V_e\right)_C = \frac{1}{2}ks_C{}^2 = \frac{1}{2}(250)(0.2^2) = 5$ J.

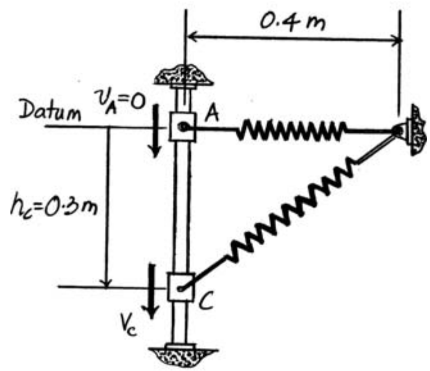
Conservation of Energy:

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}mv_A{}^2 + \left[\left(V_g\right)_A + (V_e)_A\right] = \frac{1}{2}mv_C{}^2 + \left[\left(V_g\right)_C + (V_e)_C\right]$$

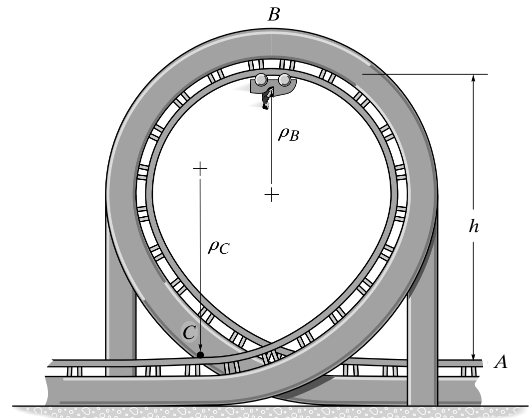
$$0 + \left(0 + 1.25\right) = \frac{1}{2}(5)v_C{}^2 + \left(-14.715 + 5\right)$$

$$v_C = 2.09 \text{ m/s}$$ **Ans.**

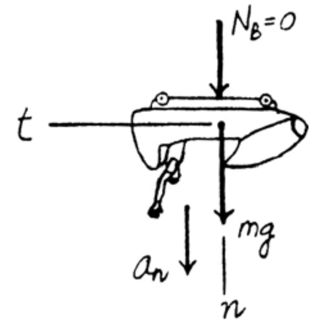


(a)

•14–89. The roller coaster and its passenger have a total mass m. Determine the smallest velocity it must have when it enters the loop at A so that it can complete the loop and not leave the track. Also, determine the normal force the tracks exert on the car when it comes around to the bottom at C. The radius of curvature of the tracks at B is ρ_B, and at C it is ρ_C. Neglect the size of the car. Points A and C are at the same elevation.



Equations of Motion: In order for the roller coaster to just pass point B without falling off the track, it is required that $N_B = 0$. Applying Eq. 13–8, we have



$$\Sigma F_n = ma_n; \qquad mg = m\left(\frac{v_B^2}{\rho_B}\right) \qquad v_B^2 = \rho_B\, g$$

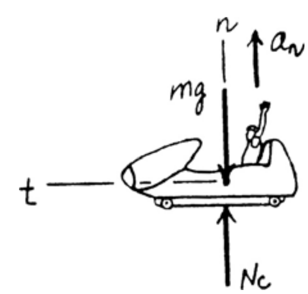
Potential Energy: Datum is set at lowest point A. When the roller coaster is at point B, its position is h *above* the datum. Thus, the gravitational potential energy at this point is mgh.

Conservation of Energy: When the roller coaster travels from A to B, we have

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A^2 + 0 = \frac{1}{2}m(\rho_B\, g) + mgh$$

$$v_A = \sqrt{\rho_B\, g + 2gh} \qquad \textbf{Ans.}$$



When the roller coaster travels from A to C, we have

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}m(\rho_B\, g + 2gh) + 0 = \frac{1}{2}mv_C^2 + 0$$

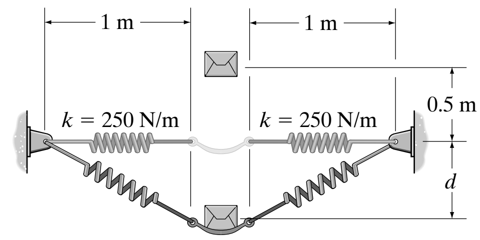
$$v_C^2 = \rho_B\, g + 2gh$$

Equations of Motion:

$$\Sigma F_n = ma_n; \qquad N_C - mg = m\left(\frac{\rho_B\, g + 2gh}{\rho_C}\right)$$

$$N_C = \frac{mg}{\rho_C}(\rho_B + \rho_C + 2h) \qquad \textbf{Ans.}$$

14–94. A pan of negligible mass is attached to two identical springs of stiffness $k = 250$ N/m. If a 10-kg box is dropped from a height of 0.5 m above the pan, determine the maximum vertical displacement d. Initially each spring has a tension of 50 N.



Potential Energy: With reference to the datum set in Fig. a, the gravitational potential energy of the box at positions (1) and (2) are $\left(V_g\right)_1 = mgh_1 = 10(9.81)(0) = 0$ and $\left(V_g\right)_2 = mgh_2 = 10(9.81)\left[-\left(0.5 + d\right)\right] = -98.1\left(0.5 + d\right)$. Initially, the spring stretches $s_1 = \dfrac{50}{250} = 0.2$ m. Thus, the unstretched length of the spring is $l_0 = 1 - 0.2 = 0.8$ m and the initial elastic potential of each spring is $\left(V_e\right)_1 = (2)\dfrac{1}{2}ks_1{}^2 = 2(250\,/\,2)(0.2^2) = 10$ J. When the box is at position (2), the spring stretches $s_2 = \left(\sqrt{d^2 + 1^2} - 0.8\right)$ m. The elastic potential energy of the springs when the box is at this position is

$$\left(V_e\right)_2 = (2)\frac{1}{2}ks_2{}^2 = 2(250\,/\,2)\left[\sqrt{d^2 + 1} - 0.8\right]^2 = 250\left(d^2 - 1.6\sqrt{d^2 + 1} + 1.64\right).$$

Conservation of Energy:

$$T_1 + V_1 + T_2 + V_2$$

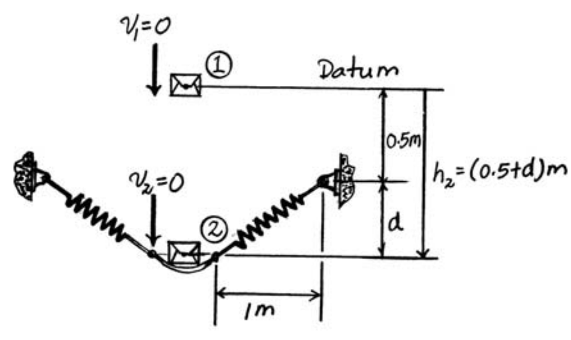
$$\frac{1}{2}mv_1{}^2 + \left[\left(V_g\right)_1 + \left(V_e\right)_1\right] = \frac{1}{2}mv_2{}^2 + \left[\left(V_g\right)_2 + \left(V_e\right)_2\right]$$

$$0 + \left(0 + 10\right) = 0 + \left[-98.1\left(0.5 + d\right) + 250\left(d^2 - 1.6\sqrt{d^2 + 1} + 1.64\right)\right]$$

$$250d^2 - 98.1d - 400\sqrt{d^2 + 1} + 350.95 = 0$$

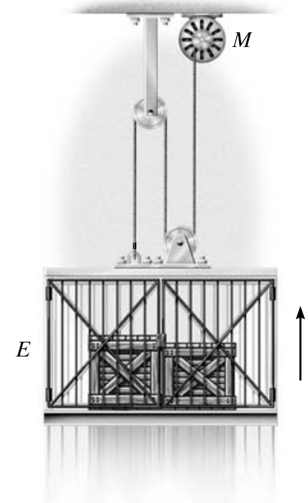
Solving the above equation by trial and error,

$$d = 1.34 \text{ m}$$ **Ans.**



(a)

14–65. The 500-kg elevator starts from rest and travels upward with a constant acceleration $a_c = 2\ \text{m/s}^2$. Determine the power output of the motor M when $t = 3$ s. Neglect the mass of the pulleys and cable.



$$+\uparrow \Sigma F_y = m\, a_y; \qquad 3T - 500(9.81) = 500(2)$$

$$T = 1968.33\ \text{N}$$

$$3s_E - s_P = l$$

$$3\, v_E = v_P$$

When $t = 3$ s,

$$(+\uparrow)\ v_0 + a_c\, t$$

$$v_E = 0 + 2(3) = 6\ \text{m/s}$$

$$v_P = 3(6) = 18\ \text{m/s}$$

$$P_O = 1968.33(18)$$

$$P_O = 35.4\ \text{kW}$$ **Ans.**