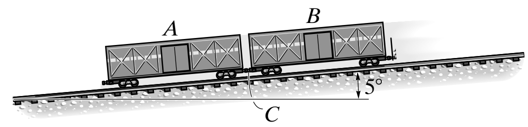
**•13–13.** The two boxcars $A$ and $B$ have a weight of 20 000 lb and 30 000 lb, respectively. If they coast freely down the incline when the brakes are applied to all the wheels of car $A$ causing it to skid, determine the force in the coupling $C$ between the two cars. The coefficient of kinetic friction between the wheels of $A$ and the tracks is $\mu_k = 0.5$. The wheels of car $B$ are free to roll. Neglect their mass in the calculation. *Suggestion:* Solve the problem by representing single resultant normal forces acting on $A$ and $B$, respectively.




Car $A$:

$+\nwarrow\Sigma F_y = 0; \qquad N_A - 20\,000 \cos 5° = 0 \qquad N_A = 19\,923.89 \text{ lb}$

$$+\nearrow\Sigma F_x = ma_x; \qquad 0.5(19\,923.89) - T - 20\,000 \sin 5° = \left(\frac{20\,000}{32.2}\right)a \qquad \textbf{(1)}$$

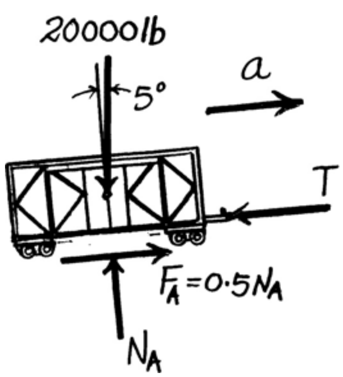
Both cars:

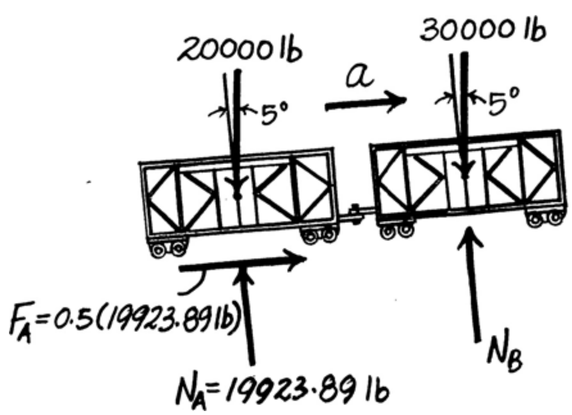
$$+\nearrow\Sigma F_x = ma_x; \qquad 0.5(19\,923.89) - 50\,000 \sin 5° = \left(\frac{50\,000}{32.2}\right)a$$

Solving,

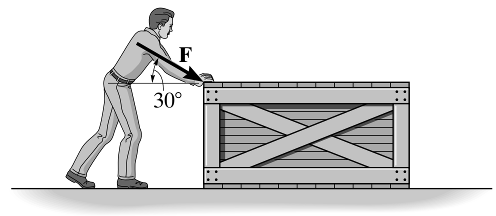
$a = 3.61 \text{ ft/s}^2$

$T = 5.98 \text{ kip}$ **Ans.**



***13–16.** The man pushes on the 60-lb crate with a force **F**. The force is always directed down at 30° from the horizontal as shown, and its magnitude is increased until the crate begins to slide. Determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.6$ and the coefficient of kinetic friction is $\mu_k = 0.3$.




Force to produce motion:

$\xrightarrow{+} \Sigma F_x = 0;\qquad F\cos 30^\circ - 0.6N = 0$

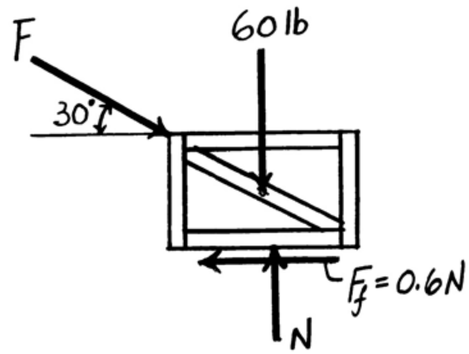
$+\uparrow \Sigma F_y = 0;\qquad N - 60 - F\sin 30^\circ = 0$

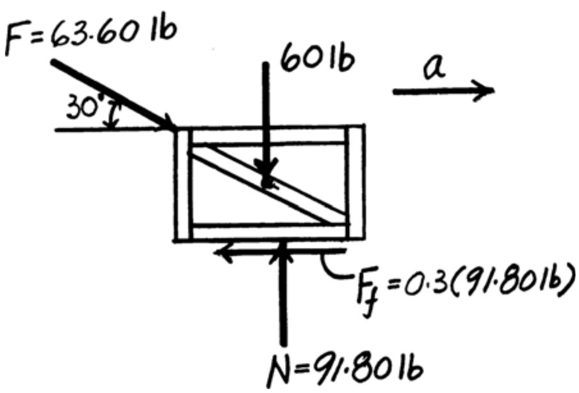
$N = 91.80 \text{ lb} \qquad F = 63.60 \text{ lb}$

Since $N = 91.80$ lb,

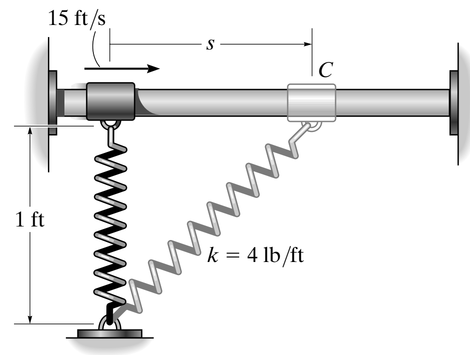
$\xrightarrow{+} \Sigma F_x = ma_x;\qquad 63.60\cos 30^\circ - 0.3(91.80) = \left(\frac{60}{32.2}\right)a$

$a = 14.8 \text{ ft/s}^2$ **Ans.**



**•13–33.** The 2-lb collar $C$ fits loosely on the smooth shaft. If the spring is unstretched when $s = 0$ and the collar is given a velocity of 15 ft/s, determine the velocity of the collar when $s = 1$ ft.



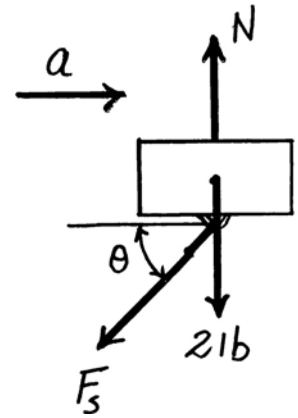

$F_s = kx; \qquad F_s = 4\left(\sqrt{1 + s^2} - 1\right)$

$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -4\left(\sqrt{1 + s^2} - 1\right)\left(\dfrac{s}{\sqrt{1 + s^2}}\right) = \left(\dfrac{2}{32.2}\right)\left(v\dfrac{dv}{ds}\right)$

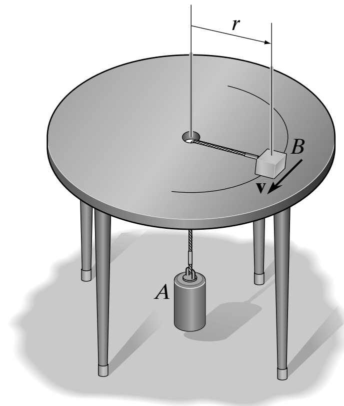
$-\displaystyle\int_0^1 \left(4s\,ds - \dfrac{4s\,ds}{\sqrt{1 + s^2}}\right) = \int_{15}^{v} \left(\dfrac{2}{32.2}\right) v\,dv$

$-\left[2s^2 - 4\sqrt{1 + s^2}\right]_0^1 = \dfrac{1}{32.2}\left(v^2 - 15^2\right)$

$v = 14.6$ ft/s **Ans.**



•**13–49.** The 2-kg block $B$ and 15-kg cylinder $A$ are connected to a light cord that passes through a hole in the center of the smooth table. If the block travels along a circular path of radius $r = 1.5\,\mathrm{m}$, determine the speed of the block.

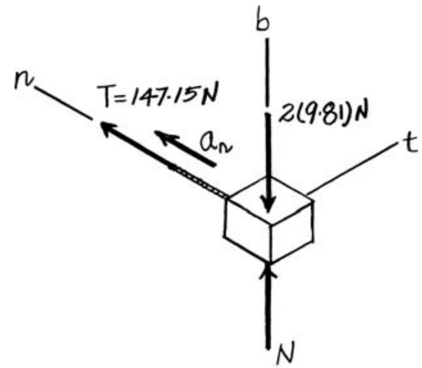



**Free-Body Diagram:** The free-body diagram of block $B$ is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder $A$, i.e., $T = 15(9.81)\mathrm{N} = 147.15\mathrm{N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive $n$ axis).

**Equations of Motion:** Realizing that $a_n = \dfrac{v^2}{r} = \dfrac{v^2}{1.5}$ and referring to Fig. (a),

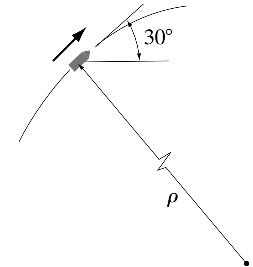
$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{v^2}{1.5}\right)$$

$$v = 10.5\ \mathrm{m/s}$$ **Ans.**




(a)

**13–51.** At the instant shown, the radius of curvature of the vertical trajectory of the 50-kg projectile is $\rho = 200\,\text{m}$. Determine the speed of the projectile at this instant.

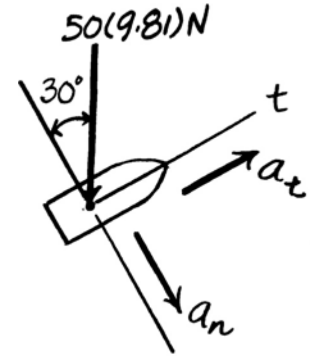



**Free-Body Diagram:** The free-body diagram of the projectile is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the trajectory (positive $n$ axis).

**Equations of Motion:** Here, $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{200}$. By referring to Fig. (a),

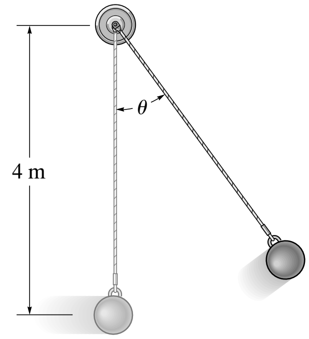
$$\searrow + \Sigma F_n = ma_n; \qquad 50(9.81)\cos 30^\circ = 50\left(\frac{v^2}{200}\right)$$

$$v = 41.2\ \text{m/s}$$ **Ans.**



(a)

**13–62.** The ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$. Determine the tension in the cord and the rate at which the ball's speed is decreasing at the instant $\theta = 20°$. Neglect the size of the ball.




$+\nwarrow \Sigma F_n = ma_n; \qquad T - 30(9.81)\cos\theta = 30\left(\frac{v^2}{4}\right)$

$+\nearrow \Sigma F_t = ma_t; \qquad -30(9.81)\sin\theta = 30a_t$

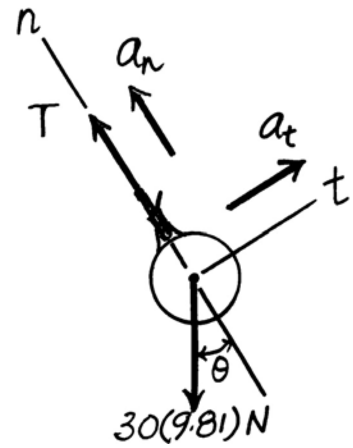
$a_t = -9.81 \sin\theta$

$a_t\, ds = v\, dv$ Since $ds = 4\, d\theta$, then

$$-9.81\int_0^\theta \sin\theta\,(4\,d\theta) = \int_4^v v\,dv$$

$$9.81(4)\cos\theta\Big|_0^\theta = \frac{1}{2}(v)^2 - \frac{1}{2}(4)^2$$

$$39.24(\cos\theta - 1) + 8 = \frac{1}{2}v^2$$



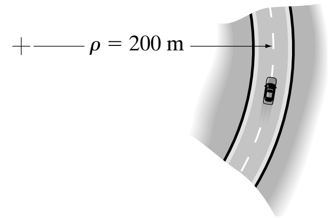

At $\theta = 20°$

$v = 3.357$ m/s

$a_t = -3.36\text{ m/s}^2 = 3.36\text{ m/s}^2 \swarrow$ **Ans.**

$T = 361$ N **Ans.**

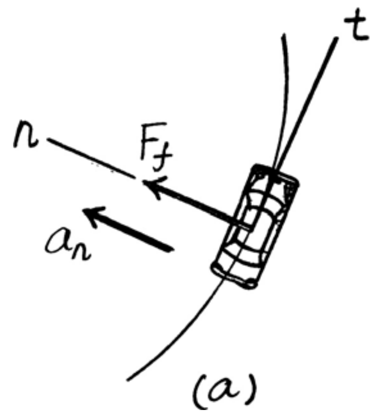
**13–67.** If the coefficient of static friction between the tires and the road surface is $\mu_s = 0.25$, determine the maximum speed of the 1.5-Mg car without causing it to slide when it travels on the curve. Neglect the size of the car.




**Free-Body Diagram:** The frictional force $\mathbf{F}_f$ developed between the tires and the road surface and $\mathbf{a}_n$ must be directed towards the center of curvature (positive $n$ axis) as indicated on the free-body diagram of the car, Fig. (a).

**Equations of Motion:** Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{200}$ and referring to Fig. (a),

$$+\nwarrow \Sigma F_n = ma_n; \qquad F_f = 1500\left(\frac{v^2}{200}\right) = 7.5v^2$$



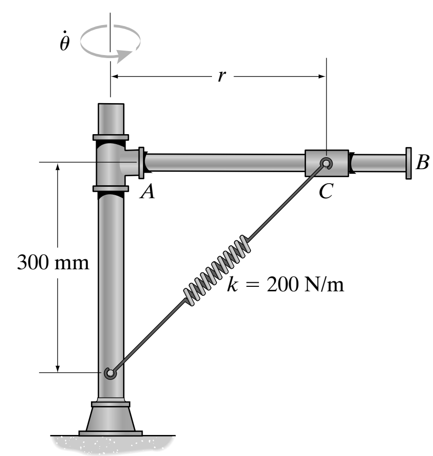

The normal reaction acting on the car is equal to the weight of the car, i.e., $N = 1500(9.81) = 14\,715$ N. When the car is on the verge of sliding,

$$F_f = \mu_s N$$

$$7.5\,v^2 = 0.25(14\,715)$$

$$v = 22.1 \text{ m/s}$$ **Ans.**

**13–94.** If the position of the 3-kg collar $C$ on the smooth rod $AB$ is held at $r = 720$ mm, determine the constant angular velocity $\dot{\theta}$ at which the mechanism is rotating about the vertical axis. The spring has an unstretched length of 400 mm. Neglect the mass of the rod and the size of the collar.




**Free-Body Diagram:** The free-body diagram of the collar is shown in Fig. (a). The force in the spring is given by $F_{sp} = ks = 200\left(\sqrt{0.72^2 + 0.3^2} - 0.4\right) = 76$ N. Here, $\mathbf{a}_r$ is assumed to be directed towards the positive $r$ axis.

**Equations of Motion:** By referring to Fig. (a),

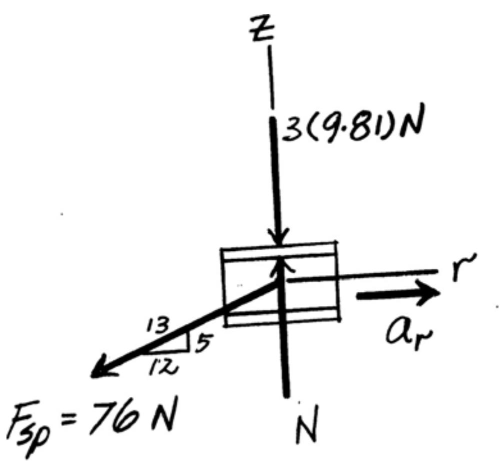
$$\xrightarrow{+} \Sigma F_r = ma_r; \qquad -76\left(\frac{12}{13}\right) = 3a_r \qquad a_r = -23.38 \text{ m/s}^2$$

**Kinematics:** Since $r = 0.72$ m is constant, $\dot{r} = \ddot{r} = 0$.

$$a_r = \ddot{r} - r\dot{\theta}^2$$

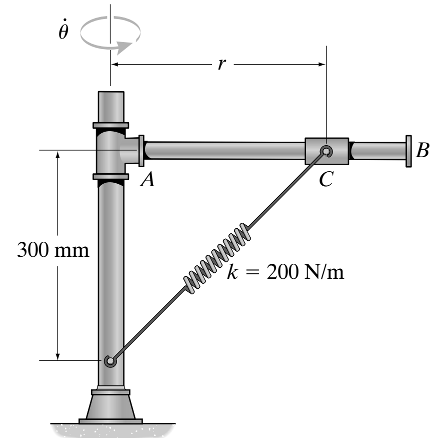
$$-23.38 = 0 - 0.72\dot{\theta}^2$$

$$\dot{\theta} = 5.70 \text{ rad/s}$$ **Ans.**




(a)

**13–95.** The mechanism is rotating about the vertical axis with a constant angular velocity of $\dot{\theta} = 6\,\text{rad/s}$. If rod $AB$ is smooth, determine the constant position $r$ of the 3-kg collar $C$. The spring has an unstretched length of 400 mm. Neglect the mass of the rod and the size of the collar.




**Free-Body Diagram:** The free-body diagram of the collar is shown in Fig. (a). The force in the spring is given by $F_{sp} = ks = 200\left(\sqrt{r^2 + 0.3^2} - 0.4\right)$. Here, $\mathbf{a}_r$ is assumed to be directed towards the positive $r$ axis.

**Equations of Motion:** By referring to Fig. (a),

$$\xrightarrow{+} \Sigma F_r = ma_r; \qquad -200\left(\sqrt{r^2 + 0.3^2} - 0.4\right)\cos\alpha = 3a_r \qquad \textbf{(1)}$$

However, from the geometry shown in Fig. (b),

$$\cos\alpha = \frac{r}{\sqrt{r^2 + 0.3^2}}$$

Thus, Eq. (1) can be rewritten as

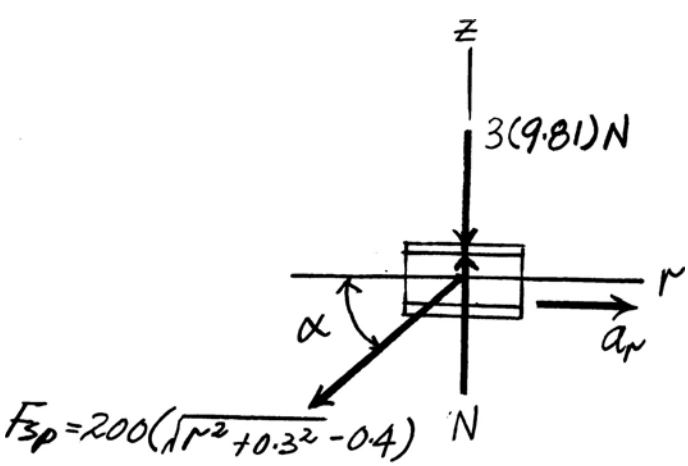
$$-200\left(r - \frac{0.4r}{\sqrt{r^2 + 0.3^2}}\right) = 3a_r \qquad \textbf{(2)}$$

**Kinematics:** Since $r$ is constant, $\dot{r} = \ddot{r} = 0$.

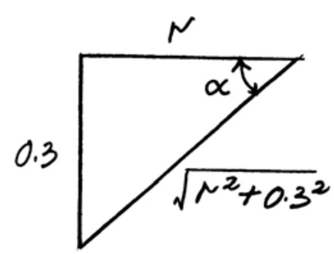
$$a_r = \ddot{r} - r\dot{\theta}^2 = -r(6^2) \qquad \textbf{(3)}$$

Substituting Eq. (3) into Eq. (2) and solving,

$$r = 0.8162\text{ m} = 816\text{ mm} \qquad \textbf{Ans.}$$

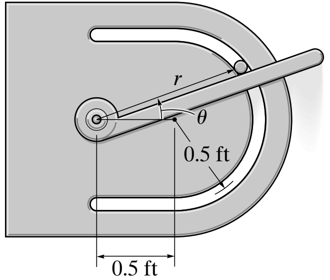



(a)




(b)

•**13–97.** The 0.75-lb smooth can is guided along the circular path using the arm guide. If the arm has an angular velocity $\dot{\theta} = 2$ rad/s and an angular acceleration $\ddot{\theta} = 0.4$ rad/s$^2$ at the instant $\theta = 30°$, determine the force of the guide on the can. Motion occurs in the *horizontal plane.*




$$r = \cos\theta\big|_{\theta=30°} = 0.8660 \text{ ft}$$

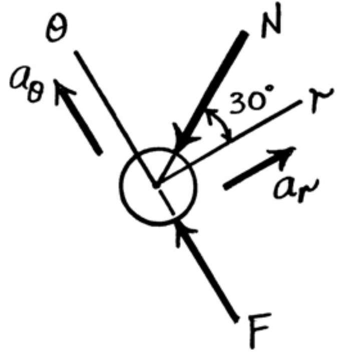
$$\dot{r} = -\sin\theta\dot{\theta}\big|_{\theta=30°} = -1.00 \text{ ft/s}$$

$$\ddot{r} = -(\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta})\big|_{\theta=30°} = -3.664 \text{ ft/s}^2$$

Using the above time derivative, we obtain

$$a_r = \ddot{r} - r\dot{\theta}^2 = -3.664 - 0.8660\left(2^2\right) = -7.128 \text{ ft/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.8660(4) + 2(-1)(2) = -0.5359 \text{ ft/s}^2$$

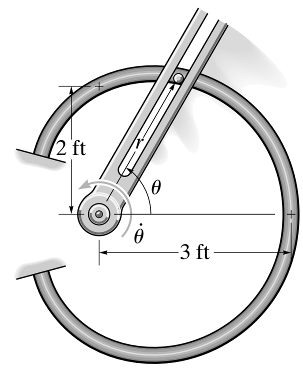



**Equations of Motion:** By referring to Fig. (a),

$\Sigma F_r = ma_r;\qquad -N\cos 30° = \dfrac{0.75}{32.2}(-7.128)\qquad N = 0.1917 \text{ lb}$

$\Sigma F_\theta = ma_\theta;\qquad F - 0.1917\sin 30° = \dfrac{0.75}{32.2}(-0.5359)\qquad F = 0.0835 \text{ lb}\qquad$ **Ans.**

**•13–101.** The forked rod is used to move the smooth 2-lb particle around the horizontal path in the shape of a limaçon, $r = (2 + \cos\theta)$ ft. If $\theta = (0.5t^2)$ rad, where $t$ is in seconds, determine the force which the rod exerts on the particle at the instant $t = 1$ s. The fork and path contact the particle on only one side.




$r = 2 + \cos\theta \qquad \theta = 0.5t^2$

$\dot{r} = -\sin\theta\theta \qquad \dot{\theta} = t$

$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta} \qquad \ddot{\theta} = 1 \text{ rad/s}^2$

At $t = 1$ s, $\theta = 0.5$ rad, $\theta = 1$ rad/s, and $\ddot{\theta} = 1 \text{ rad/s}^2$

$r = 2 + \cos 0.5 = 2.8776$ ft

$\dot{r} = -\sin 0.5(1) = -0.4974 \text{ ft/s}^2$

$\ddot{r} = -\cos 0.5(1)^2 - \sin 0.5(1) = -1.357 \text{ ft/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -1.375 - 2.8776(1)^2 = -4.2346 \text{ ft/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.8776(1) + 2(-0.4794)(1) = 1.9187 \text{ ft/s}^2$

$$\tan\psi = \frac{r}{dr/d\theta} = \left.\frac{2 + \cos\theta}{-\sin\theta}\right|_{\theta = 0.5 \text{ rad}} = -6.002 \qquad \psi = -80.54^\circ$$

$$+\nearrow\Sigma F_r = ma_r; \qquad -N\cos 9.46^\circ = \frac{2}{32.2}(-4.2346) \qquad N = 0.2666 \text{ lb}$$

$$+\nwarrow\Sigma F_\theta = ma_\theta; \qquad F - 0.2666\sin 9.46^\circ = \frac{2}{32.2}(1.9187)$$

$F = 0.163$ lb **Ans.**