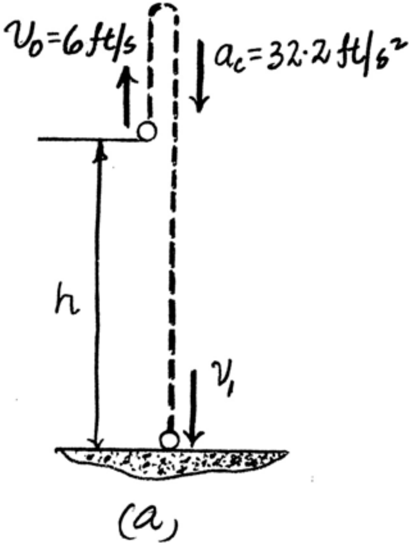
12–6. A ball is released from the bottom of an elevator which is traveling upward with a velocity of 6 ft/s. If the ball strikes the bottom of the elevator shaft in 3 s, determine the height of the elevator from the bottom of the shaft at the instant the ball is released. Also, find the velocity of the ball when it strikes the bottom of the shaft.

Kinematics: When the ball is released, its velocity will be the same as the elevator at the instant of release. Thus, $v_0 = 6$ ft/s. Also, $t = 3$ s, $s_0 = 0$, $s = -h$, and $a_c = -32.2$ ft/s^2.



$(+\uparrow)$ $$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$-h = 0 + 6(3) + \frac{1}{2}(-32.2)(3^2)$$

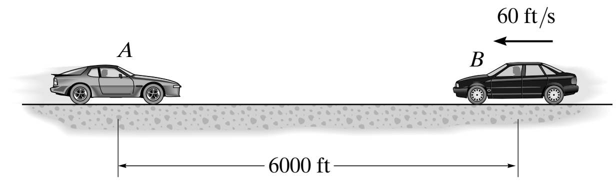
$$h = 127 \text{ ft}$$ **Ans.**

$(+\uparrow)$ $$v = v_0 + a_c t$$

$$v = 6 + (-32.2)(3)$$

$$= -90.6 \text{ ft/s} = 90.6 \text{ ft/s} \downarrow$$ **Ans.**

12–10. Car A starts from rest at $t = 0$ and travels along a straight road with a constant acceleration of 6 ft/s^2 until it reaches a speed of 80 ft/s. Afterwards it maintains this speed. Also, when $t = 0$, car B located 6000 ft down the road is traveling towards A at a constant speed of 60 ft/s. Determine the distance traveled by car A when they pass each other.



Distance Traveled: Time for car A to achives $v = 80$ ft/s can be obtained by applying Eq. 12–4.

$$(\overset{+}{\rightarrow}) \qquad v = v_0 + a_c t$$

$$80 = 0 + 6t$$

$$t = 13.33 \text{ s}$$

The distance car A travels for this part of motion can be determined by applying Eq. 12–6.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$80^2 = 0 + 2(6)(s_1 - 0)$$

$$s_1 = 533.33 \text{ ft}$$

For the second part of motion, car A travels with a constant velocity of $v = 80$ ft/s and the distance traveled in $t' = (t_1 - 13.33)$ s (t_1 is the total time) is

$$(\overset{+}{\rightarrow}) \qquad s_2 = vt' = 80(t_1 - 13.33)$$

Car B travels in the opposite direction with a constant velocity of $v = 60$ ft/s and the distance traveled in t_1 is

$$(\overset{+}{\rightarrow}) \qquad s_3 = vt_1 = 60t_1$$

It is required that

$$s_1 + s_2 + s_3 = 6000$$

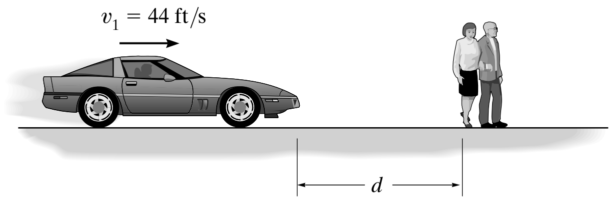
$$533.33 + 80(t_1 - 13.33) + 60t_1 = 6000$$

$$t_1 = 46.67 \text{ s}$$

The distance traveled by car A is

$$s_A = s_1 + s_2 = 533.33 + 80(46.67 - 13.33) = 3200 \text{ ft}$$ **Ans.**

12–15. Tests reveal that a normal driver takes about 0.75 s before he or she can *react* to a situation to avoid a collision. It takes about 3 s for a driver having 0.1% alcohol in his system to do the same. If such drivers are traveling on a straight road at 30 mph (44 ft/s) and their cars can decelerate at 2 ft/s^2, determine the shortest stopping distance d for each from the moment they see the pedestrians. *Moral*: If you must drink, please don't drive!



Stopping Distance: For normal driver, the car moves a distance of $d' = vt = 44(0.75) = 33.0$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 33.0$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = 44^2 + 2(-2)(d - 33.0)$$

$$d = 517 \text{ ft} \qquad \textbf{Ans.}$$

For a drunk driver, the car moves a distance of $d' = vt = 44(3) = 132$ ft before he or she reacts and decelerates the car. The stopping distance can be obtained using Eq. 12–6 with $s_0 = d' = 132$ ft and $v = 0$.

$$(\overset{+}{\rightarrow}) \qquad v^2 = v_0^2 + 2a_c(s - s_0)$$

$$0^2 = 44^2 + 2(-2)(d - 132)$$

$$d = 616 \text{ ft} \qquad \textbf{Ans.}$$

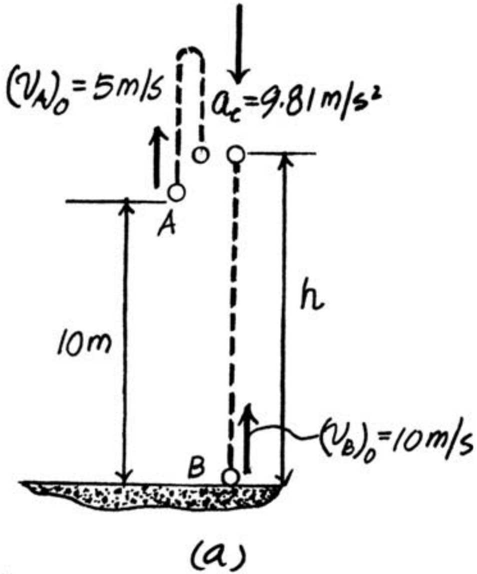
•12–17. A ball is thrown with an upward velocity of 5 m/s from the top of a 10-m high building. One second later another ball is thrown vertically from the ground with a velocity of 10 m/s. Determine the height from the ground where the two balls pass each other.

Kinematics: First, we will consider the motion of ball A with $(v_A)_0 = 5$ m/s, $(s_A)_0 = 0$, $s_A = (h - 10)$ m, $t_A = t'$, and $a_c = -9.81$ m/s^2. Thus,

$$(+\uparrow) \qquad s_A = (s_A)_0 + (v_A)_0 t_A + \frac{1}{2} a_c t_A{}^2$$

$$h - 10 = 0 + 5t' + \frac{1}{2}(-9.81)(t')^2$$

$$h = 5t' - 4.905(t')^2 + 10 \qquad \textbf{(1)}$$



Motion of ball B is with $(v_B)_0 = 10$ m/s, $(s_B)_0 = 0$, $s_B = h$, $t_B = t' - 1$ and $a_c = -9.81$ m/s^2. Thus,

$$(+\uparrow) \qquad s_B = (s_B)_0 + (v_B)_0 t_B + \frac{1}{2} a_c t_B{}^2$$

$$h = 0 + 10(t' - 1) + \frac{1}{2}(-9.81)(t' - 1)^2$$

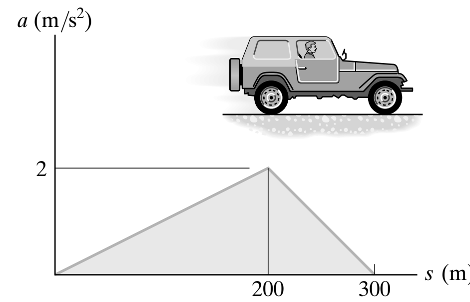
$$h = 19.81t' - 4.905(t')^2 - 14.905 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$h = 4.54 \text{ m} \qquad \textbf{Ans.}$$

$$t' = 1.68 \text{ m}$$

***12–48.** The a–s graph for a jeep traveling along a straight road is given for the first 300 m of its motion. Construct the v–s graph. At $s = 0, v = 0$.

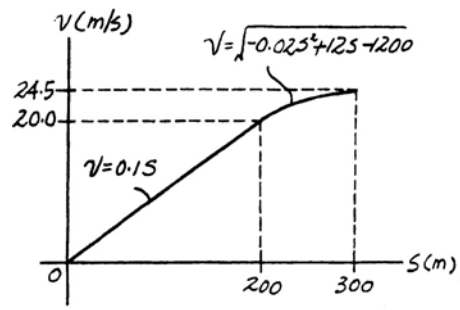


$a-s$ Graph: The function of acceleration a in terms of s for the interval $0 \text{ m} \leq s < 200 \text{ m}$ is

$$\frac{a-0}{s-0} = \frac{2-0}{200-0} \qquad a = (0.01s) \text{ m/s}^2$$

For the interval $200 \text{ m} < s \leq 300 \text{ m}$,

$$\frac{a-2}{s-200} = \frac{0-2}{300-200} \qquad a = (-0.02s + 6) \text{ m/s}^2$$



$v-s$ Graph: The function of velocity v in terms of s can be obtained by applying $vdv = ads$. For the interval **$0 \text{ m} \leq s < 200 \text{ m}$**,

$$vdv = ds$$

$$\int_0^v vdv = \int_0^s 0.01sds$$

$$v = (0.1s) \text{ m/s}$$

At $s = 200$ m, $\qquad v = 0.100(200) = 20.0 \text{ m/s}$

For the interval **$200 \text{ m} < s \leq 300 \text{ m}$**,

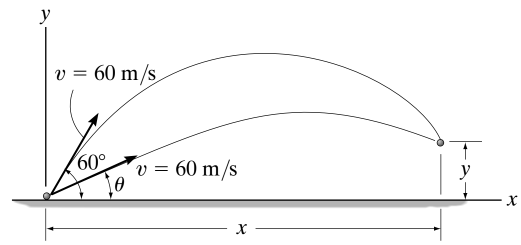
$$vdv = ads$$

$$\int_{20.0\text{m/s}}^v vdv = \int_{200m}^s (-0.02s + 6)ds$$

$$v = \left(\sqrt{-0.02s^2 + 12s - 1200}\right) \text{ m/s}$$

At $s = 300$ m, $\qquad v = \sqrt{-0.02(300^2) + 12(300) - 1200} = 24.5 \text{ m/s}$

12–90. A projectile is fired with a speed of $v = 60$ m/s at an angle of 60°. A second projectile is then fired with the same speed 0.5 s later. Determine the angle θ of the second projectile so that the two projectiles collide. At what position (x, y) will this happen?



***x*-Motion:** For the motion of the first projectile, $v_x = 60 \cos 60° = 30$ m/s, $x_0 = 0$, and $t = t_1$. Thus,

$$(\overset{+}{\rightarrow}) \qquad x = x_0 + v_x t$$

$$x = 0 + 30t_1 \qquad \textbf{(1)}$$

For the motion of the second projectile, $v_x = 60 \cos \theta$, $x_0 = 0$, and $t = t_1 - 0.5$. Thus,

$$(\overset{+}{\rightarrow}) \qquad x = x_0 + v_x t$$

$$x = 0 + 60 \cos \theta (t_1 - 0.5) \qquad \textbf{(2)}$$

***y*-Motion:** For the motion of the first projectile, $v_y = 60 \sin 60° = 51.96$ m/s, $y_0 = 0$, and $a_y = -g = -9.81 \text{ m/s}^2$. Thus,

$$(+\uparrow) \qquad y = y_0 + v_y t + \frac{1}{2} a_y t^2$$

$$y = 0 + 51.96t_1 + \frac{1}{2}(-9.81)t_1^{\,2}$$

$$y = 51.96t_1 - 4.905t_1^{\,2} \qquad \textbf{(3)}$$

For the motion of the second projectile, $v_y = 60 \sin \theta$, $y_0 = 0$, and $a_y = -g = -9.81 \text{ m/s}^2$. Thus,

$$(+\uparrow) \qquad y = y_0 + v_y t + \frac{1}{2} a_y t^2$$

$$y = 0 + 60 \sin \theta (t_1 - 0.5) + \frac{1}{2}(-9.81)(t_1 - 0.5)^2$$

$$y = (60 \sin \theta)t_1 - 30 \sin \theta - 4.905\, t_1^{\,2} + 4.905t_1 - 1.22625 \qquad \textbf{(4)}$$

Equating Eqs. (1) and (2),

$$30t_1 = 60 \cos \theta (t_1 - 0.5)$$

$$t_1 = \frac{\cos \theta}{2 \cos \theta - 1} \qquad \textbf{(5)}$$

Equating Eqs. (3) and (4),

$$51.96t_1 - 4.905t_1^{\,2} = (60 \sin \theta)t_1 - 30 \sin \theta - 4.905t_1^{\,2} + 4.905t_1 - 1.22625$$

$$(60 \sin \theta - 47.06)t_1 = 30 \sin \theta + 1.22625$$

$$t_1 = \frac{30 \sin \theta + 1.22625}{60 \sin \theta - 47.06} \qquad \textbf{(6)}$$

12–90. Continued

Equating Eqs. (5) and (6) yields

$$\frac{\cos\theta}{2\cos\theta - 1} = \frac{30\sin\theta + 1.22625}{60\sin\theta - 47.06}$$

$$49.51\cos\theta - 30\sin\theta = 1.22625$$

Solving by trial and error,

$$\theta = 57.57° = 57.6°$$ **Ans.**

Substituting this result into Eq. (5) (or Eq. (6)),

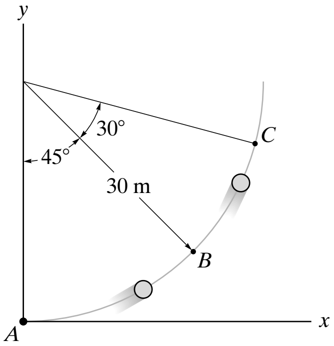
$$t_1 = \frac{\cos 57.57°}{2\cos 57.57° - 1} = 7.3998 \text{ s}$$

Substituting this result into Eqs. (1) and (3),

$$x = 30(7.3998) = 222 \text{ m}$$ **Ans.**

$$y = 51.96(7.3998) - 4.905\left(7.3998^2\right) = 116 \text{ m}$$ **Ans.**

•**12–81.** A particle travels along the circular path from *A* to *B* in 1 s. If it takes 3 s for it to go from *A* to *C*, determine its *average velocity* when it goes from *B* to *C*.



Position: The coordinates for points B and C are $[30 \sin 45°, 30 - 30 \cos 45°]$ and $[30 \sin 75°, 30 - 30 \cos 75°]$. Thus,

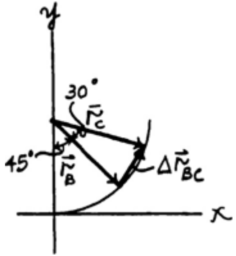
$$\mathbf{r}_B = (30 \sin 45° - 0)\mathbf{i} + [(30 - 30 \cos 45°) - 30]\mathbf{j}$$

$$= \{21.21\mathbf{i} - 21.21\mathbf{j}\} \text{ m}$$

$$\mathbf{r}_C = (30 \sin 75° - 0)\mathbf{i} + [(30 - 30 \cos 75°) - 30]\mathbf{j}$$

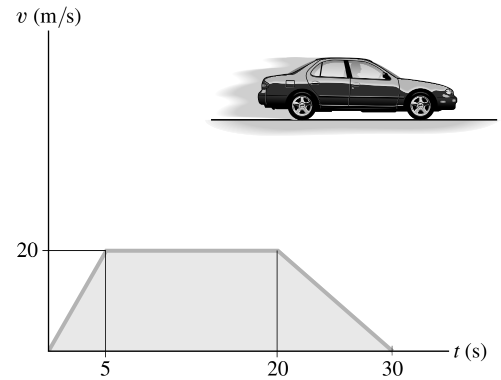
$$= \{28.98\mathbf{i} - 7.765\mathbf{j}\} \text{ m}$$

Average Velocity: The displacement from point *B* to *C* is $\Delta\mathbf{r}_{BC} = \mathbf{r}_C - \mathbf{r}_B = (28.98\mathbf{i} - 7.765\mathbf{j}) - (21.21\mathbf{i} - 21.21\mathbf{j}) = \{7.765\mathbf{i} + 13.45\mathbf{j}\}$ m.



$$(\mathbf{v}_{BC})_{avg} = \frac{\Delta\mathbf{r}_{BC}}{\Delta t} = \frac{7.765\mathbf{i} + 13.45\mathbf{j}}{3 - 1} = \{3.88\mathbf{i} + 6.72\mathbf{j}\} \text{ m/s} \qquad \textbf{Ans.}$$

•12–61. The $v-t$ graph of a car while traveling along a road is shown. Draw the $s-t$ and $a-t$ graphs for the motion.



$0 \leq t \leq 5$ $\quad a = \frac{\Delta v}{\Delta t} = \frac{20}{5} = 4 \text{ m/s}^2$

$5 \leq t \leq 20$ $\quad a = \frac{\Delta v}{\Delta t} = \frac{20 - 20}{20 - 5} = 0 \text{ m/s}^2$

$20 \leq t \leq 30$ $\quad a = \frac{\Delta v}{\Delta t} = \frac{0 - 20}{30 - 20} = -2 \text{ m/s}^2$

From the v–t graph at $t_1 = 5$ s, $t_2 = 20$ s, and $t_3 = 30$ s,

$s_1 = A_1 = \frac{1}{2}(5)(20) = 50 \text{ m}$

$s_2 = A_1 + A_2 = 50 + 20(20 - 5) = 350 \text{ m}$

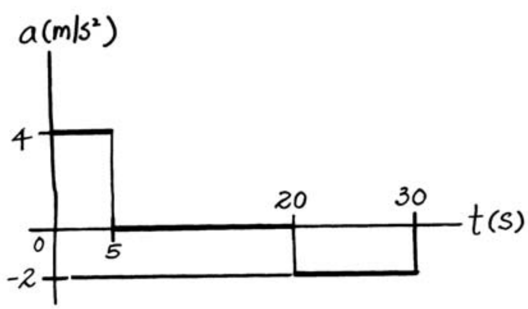
$s_3 = A_1 + A_2 + A_3 = 350 + \frac{1}{2}(30 - 20)(20) = 450 \text{ m}$

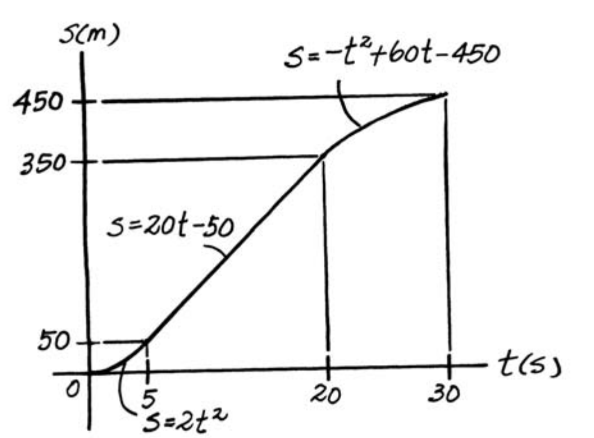
The equations defining the portions of the s–t graph are

$0 \leq t \leq 5 \text{ s}$ $\quad v = 4t; \quad ds = v\,dt; \quad \int_0^s ds = \int_0^t 4t\,dt; \quad s = 2t^2$

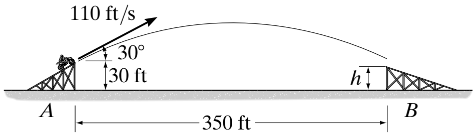
$5 \leq t \leq 20 \text{ s}$ $\quad v = 20; \quad ds = v\,dt; \quad \int_{50}^s ds = \int_5^t 20\,dt; \quad s = 20t - 50$

$20 \leq t \leq 30 \text{ s}$ $\quad v = 2(30 - t); \quad ds = v\,dt; \quad \int_{350}^s ds = \int_{20}^t 2(30 - t)\,dt; \quad s = -t^2 + 60t - 450$





12–95. If the motorcycle leaves the ramp traveling at 110 ft/s, determine the height h ramp B must have so that the motorcycle lands safely.



Coordinate System: The x–y coordinate system will be set so that its origin coincides with the take off point of the motorcycle at ramp A.

***x*-Motion:** Here, $x_A = 0$, $x_B = 350$ ft, and $(v_A)_x = 110 \cos 30° = 95.26$ ft/s. Thus,

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$350 = 0 + 95.26t$$

$$t = 3.674 \text{ s}$$

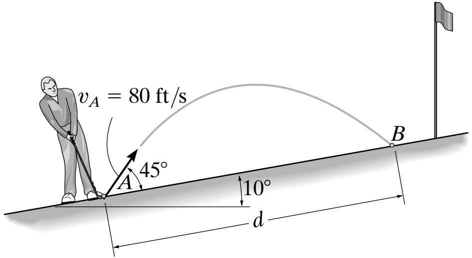
***y*-Motion:** Here, $y_A = 0$, $y_B = h - 30$, $(v_A)_y = 110 \sin 30° = 55$ ft/s, and $a_y = -g = -32.2 \text{ ft/s}^2$. Thus, using the result of t, we have

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$h - 30 = 0 + 55(3.674) + \frac{1}{2}(-32.2)\left(3.674^2\right)$$

$$h = 14.7 \text{ ft}$$ **Ans.**

12–102. A golf ball is struck with a velocity of 80 ft/s as shown. Determine the distance d to where it will land.



Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 80\cos 55° = 45.89$ ft/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = d\cos 10°$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$d\cos 10° = 0 + 45.89t \qquad \textbf{[1]}$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 80\sin 55° = 65.53$ ft/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = d\sin 10°$, respectively.

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

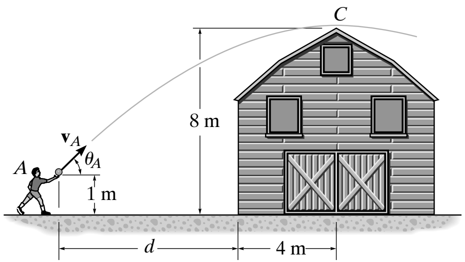
$$d\sin 10° = 0 + 65.53t + \frac{1}{2}(-32.2)t^2 \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$d = 166 \text{ ft} \qquad \textbf{Ans.}$$

$$t = 3.568 \text{ s}$$

12–106. The boy at A attempts to throw a ball over the roof of a barn such that it is launched at an angle $\theta_A = 40°$. Determine the minimum speed v_A at which he must throw the ball so that it reaches its maximum height at C. Also, find the distance d where the boy must stand so that he can make the throw.



Vertical Motion: The vertical components of initial and final velocity are $(v_0)_y = (v_A \sin 40°)$ m/s and $v_y = 0$, respectively. The initial vertical position is $(s_0)_y = 1$ m.

$(+\uparrow)$ $$v_y = (v_0) + a_c t$$

$$0 = v_A \sin 40° + (-9.81)\,t \qquad \textbf{[1]}$$

$(+\uparrow)$ $$s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$8 = 1 + v_A \sin 40° t + \frac{1}{2}(-9.81)\,t^2 \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$v_A = 18.23 \text{ m/s} = 18.2 \text{ m/s} \qquad \textbf{Ans.}$$

$$t = 1.195 \text{ s}$$

Horizontal Motion: The horizontal component of velocity is $(v_0)_x = v_A \cos\theta_A = 18.23 \cos 40° = 13.97$ m/s. The initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = (d + 4)$ m, respectively.

$(\overset{+}{\rightarrow})$ $$s_x = (s_0)_x + (v_0)_x t$$

$$d + 4 = 0 + 13.97(1.195)$$

$$d = 12.7 \text{ m} \qquad \textbf{Ans.}$$

•**12–113.** Determine the maximum constant speed a race car can have if the acceleration of the car cannot exceed 7.5 m/s^2 while rounding a track having a radius of curvature of 200 m.

Acceleration: Since the speed of the race car is constant, its tangential component of acceleration is zero, i.e., $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho}$$

$$7.5 = \frac{v^2}{200}$$

$$v = 38.7 \text{ m/s}$$ **Ans.**

12–119. A car moves along a circular track of radius 250 ft, and its speed for a short period of time $0 \leq t \leq 2$ s is $v = 3(t + t^2)$ ft/s, where t is in seconds. Determine the magnitude of the car's acceleration when $t = 2$ s. How far has it traveled in $t = 2$ s?

$$v = 3\left(t + t^2\right)$$

$$a_t = \frac{dv}{dt} = 3 + 6t$$

When $t = 2$ s,

$$a_t = 3 + 6(2) = 15 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{\left[3(2 + 2^2)\right]^2}{250} = 1.296 \text{ ft/s}^2$$

$$a = \sqrt{(15)^2 + (1.296)^2} = 15.1 \text{ ft/s}^2$$ **Ans.**

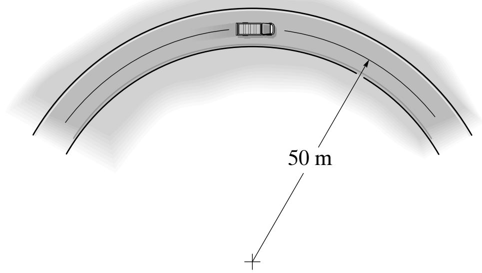
$$ds = v\,dt$$

$$\int ds = \int_0^2 3\left(t + t^2\right) dt$$

$$\Delta s = \frac{3}{2}t^2 + t^3\Big]_0^2$$

$$\Delta s = 14 \text{ ft}$$ **Ans.**

•**12–141.** The truck travels along a circular road that has a radius of 50 m at a speed of 4 m/s. For a short distance when $t = 0$, its speed is then increased by $a_t = (0.4t)\ \text{m/s}^2$, where t is in seconds. Determine the speed and the magnitude of the truck's acceleration when $t = 4$ s.



Velocity: The speed v in terms of time t can be obtained by applying $a = \dfrac{dv}{dt}$.

$$dv = adt$$

$$\int_{4\text{ m/s}}^{v} dv = \int_0^t 0.4t\,dt$$

$$v = \left(0.2t^2 + 4\right)\ \text{m/s}$$

When $t = 4$ s, $\quad v = 0.2\left(4^2\right) + 4 = 7.20\ \text{m/s}$ **Ans.**

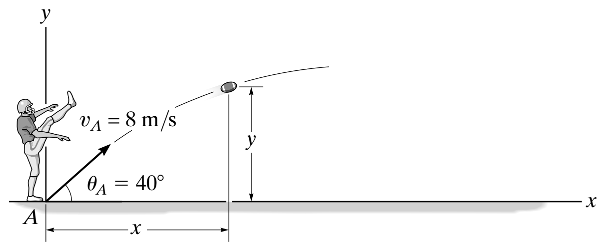
Acceleration: The tangential acceleration of the truck when $t = 4\text{ s}$ is $a_t = 0.4(4) = 1.60\ \text{m/s}^2$. To determine the normal acceleration, apply Eq. 12–20.

$$a_n = \frac{v^2}{\rho} = \frac{7.20^2}{50} = 1.037\ \text{m/s}^2$$

The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{1.60^2 + 1.037^2} = 1.91\ \text{m/s}^2$$ **Ans.**

•12–153. The ball is kicked with an initial speed $v_A = 8$ m/s at an angle $\theta_A = 40°$ with the horizontal. Find the equation of the path, $y = f(x)$, and then determine the normal and tangential components of its acceleration when $t = 0.25$ s.



Horizontal Motion: The horizontal component of velocity is $(v_0)_x = 8\cos 40° = 6.128$ m/s and the initial horizontal and final positions are $(s_0)_x = 0$ and $s_x = x$, respectively.

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + (v_0)_x t$$

$$x = 0 + 6.128t \qquad [1]$$

Vertical Motion: The vertical component of initial velocity is $(v_0)_y = 8\sin 40° = 5.143$ m/s. The initial and final vertical positions are $(s_0)_y = 0$ and $s_y = y$, respectively.

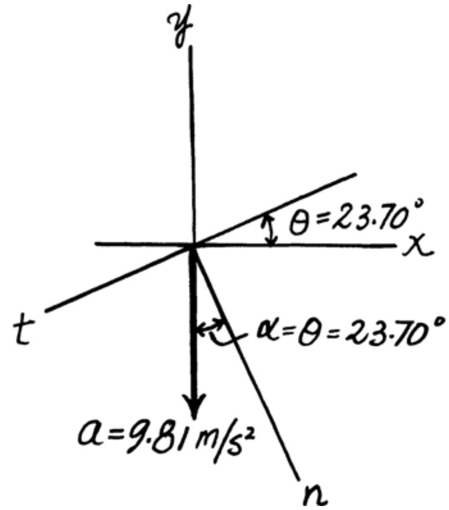
$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$y = 0 + 5.143t + \frac{1}{2}(-9.81)\left(t^2\right) \qquad [2]$$

Eliminate t from Eqs [1] and [2], we have

$$y = \{0.8391x - 0.1306x^2\}\text{ m} = \{0.839x - 0.131x^2\}\text{ m} \qquad \textbf{Ans.}$$

The tangent of the path makes an angle $\theta = \tan^{-1}\dfrac{3.644}{4} = 42.33°$ with the x axis.

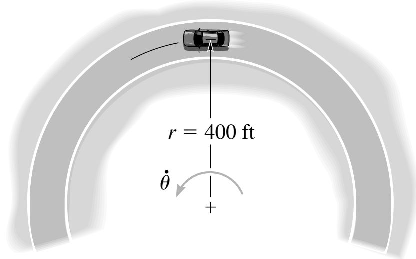


Acceleration: When $t = 0.25$ s, from Eq. [1], $x = 0 + 6.128(0.25) = 1.532$ m. Here, $\dfrac{dy}{dx} = 0.8391 - 0.2612x$. At $x = 1.532$ m, $\dfrac{dy}{dx} = 0.8391 - 0.2612(1.532) = 0.4389$ and the tangent of the path makes an angle $\theta = \tan^{-1} 0.4389 = 23.70°$ with the x axis. The magnitude of the acceleration is $a = 9.81$ m/s^2 and is directed downward. From the figure, $\alpha = 23.70°$. Therefore,

$$a_t = a\sin\alpha = 9.81\sin 23.70° = 3.94\text{ m/s}^2 \qquad \textbf{Ans.}$$

$$a_n = a\cos\alpha = 9.81\cos 23.70° = 8.98\text{ m/s}^2 \qquad \textbf{Ans.}$$

•12–169. The car travels along the circular curve of radius $r = 400$ ft with a constant speed of $v = 30$ ft/s. Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of the car's acceleration.



$r = 400 \text{ ft} \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$v_r = r = 0 \qquad v_\theta = r\theta = 400\left(\theta\right)$

$v = \sqrt{(0)^2 + \left(400\,\dot{\theta}\right)^2} = 30$

$\theta = 0.075 \text{ rad/s}$ **Ans.**

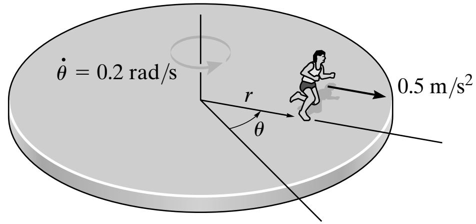
$\ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 400(0.075)^2 = -2.25 \text{ ft/s}^2$

$a_\theta = r\theta + 2\dot{r}\theta = 400(0) + 2(0)(0.075) = 0$

$a = \sqrt{(-2.25)^2 + (0)^2} = 2.25 \text{ ft/s}^2$ **Ans.**

12–170. Starting from rest, the boy runs outward in the radial direction from the center of the platform with a constant acceleration of $0.5\ \text{m/s}^2$. If the platform is rotating at a constant rate $\dot{\theta} = 0.2$ rad/s, determine the radial and transverse components of the velocity and acceleration of the boy when $t = 3$ s. Neglect his size.



Velocity: When $t = 3$ s, the position of the boy is given by

$$s = (s_0)_r + (v_0)_r\, t + \frac{1}{2}(a_c)_r\, t^2$$

$$r = 0 + 0 + \frac{1}{2}(0.5)\left(3^2\right) = 2.25\ \text{m}$$

The boy's radial component of velocity is given by

$$v_r = (v_0)_r + (a_c)_r\, t$$

$$= 0 + 0.5(3) = 1.50\ \text{m/s} \qquad \textbf{Ans.}$$

The boy's transverse component of velocity is given by

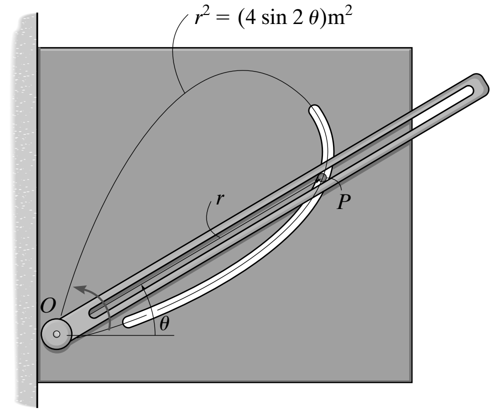
$$v_\theta = r\dot{\theta} = 2.25(0.2) = 0.450\ \text{m/s} \qquad \textbf{Ans.}$$

Acceleration: When $t = 3$ s, $r = 2.25$ m, $\dot{r} = v_r = 1.50$ m/s, $\ddot{r} = 0.5\ \text{m/s}^2$, $\ddot{\theta} = 0$. Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0.5 - 2.25\left(0.2^2\right) = 0.410\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.25(0) + 2(1.50)(0.2) = 0.600\ \text{m/s}^2 \qquad \textbf{Ans.}$$

•12–173. The peg moves in the curved slot defined by the lemniscate, and through the slot in the arm. At $\theta = 30°$, the angular velocity is $\dot{\theta} = 2$ rad/s, and the angular acceleration is $\ddot{\theta} = 1.5$ rad/s^2. Determine the magnitudes of the velocity and acceleration of peg P at this instant.



Time Derivatives:

$$2r\dot{r} = 8\cos 2\theta\dot{\theta}$$

$$\dot{r} = \left(\frac{4\cos 2\theta\dot{\theta}}{r}\right)\text{ m/s} \qquad \dot{\theta} = 2\text{ rad/s}$$

$$2\left(r\ddot{r} + \dot{r}^2\right) = 8\left(-2\sin 2\theta\dot{\theta} + \cos 2\theta\dot{\theta}^2\right)$$

$$\ddot{r} = \left[\frac{4\left(\cos 2\theta\ddot{\theta} - 2\sin 2\theta\dot{\theta}^2\right) - \dot{r}^2}{r}\right]\text{ m/s}^2 \qquad \ddot{\theta} = 1.5\text{ rad/s}^2$$

At $\theta = 30°$,

$$r\big|_{\theta=30°} = \sqrt{4\sin 60°} = 1.861\text{ m}$$

$$\dot{r}\big|_{\theta=30°} = \frac{(4\cos 60°)(2)}{1.861} = 2.149\text{ m/s}$$

$$\ddot{r}\big|_{\theta=30°} = \frac{4\left[\cos 60°(1.5) - 2\sin 60°\left(2^2\right)\right] - (2.149)^2}{1.861} = -15.76\text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = 2.149\text{ m/s} \qquad v_\theta = r\dot{\theta} = 1.861(2) = 3.722\text{ m/s}$$

Thus, the magnitude of the peg's velocity is

$$v = \sqrt{a_r{}^2 + a_\theta{}^2} = \sqrt{2.149^2 + 3.722^2} = 4.30\text{ m/s}$$ **Ans.**

Acceleration:

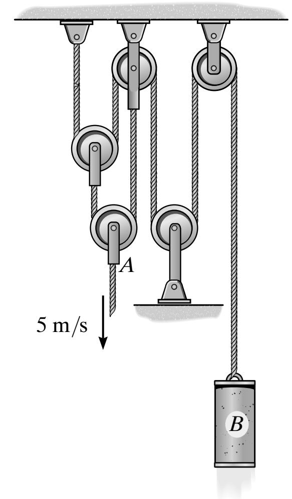
$$a_r = \ddot{r} - r\dot{\theta}^2 = -15.76 - 1.861\left(2^2\right) = -23.20\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.861(1.5) + 2(2.149)(2) = 11.39\text{ m/s}^2$$

Thus, the magnitude of the peg's acceleration is

$$a = \sqrt{a_r{}^2 + a_\theta{}^2} = \sqrt{(-23.20)^2 + 11.39^2} = 25.8\text{ m/s}^2$$ **Ans.**

12–198. If end A of the rope moves downward with a speed of $5\,\text{m/s}$, determine the speed of cylinder B.



Position Coordinates: By referring to Fig. a, the length of the two ropes written in terms of the position coordinates s_A, s_B, and s_C are

$$s_B + 2a + 2s_C = l_1$$

$$s_B + 2s_C = l_1 - 2a \qquad \textbf{(1)}$$

and

$$s_A + (s_A - s_C) = l_2$$

$$2s_A - s_C = l_2 \qquad \textbf{(2)}$$

Eliminating s_C from Eqs. (1) and (2) yields

$$s_B + 4s_A = l_1 - 2a + 2l_2$$

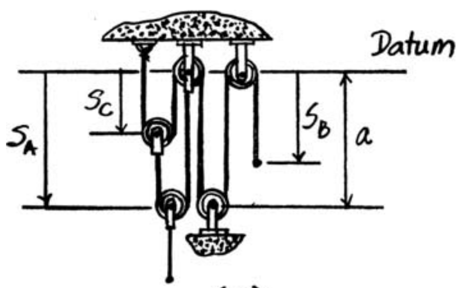
Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad v_B + 4v_A = 0$$

Here, $v_A = 5$ m/s. Thus,

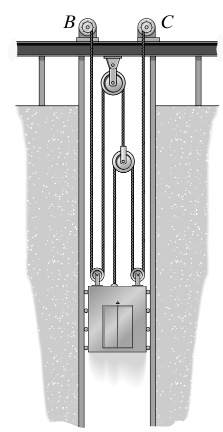
$$v_B + 4(5) = 0$$

$$v_B = -20\text{ m/s} = 20\text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(a)

12–199. Determine the speed of the elevator if each motor draws in the cable with a constant speed of $5\,\text{m/s}$.



Position Coordinates: By referring to Fig. a, the length of the two cables written in terms of the position coordinates are

$$s_E + (s_E - s_A) + s_C = l_1$$

$$2s_E - s_A + s_C = l_1 \qquad \textbf{(1)}$$

and

$$(s_E - s_B) + 2(s_E - s_C) = l_2$$

$$3s_E - s_B - 2s_C = l_2 \qquad \textbf{(2)}$$

Eliminating s_C from Eqs. (1) and (2) yields

$$7s_E - 2s_A - s_B = 2l_1 + l_2$$

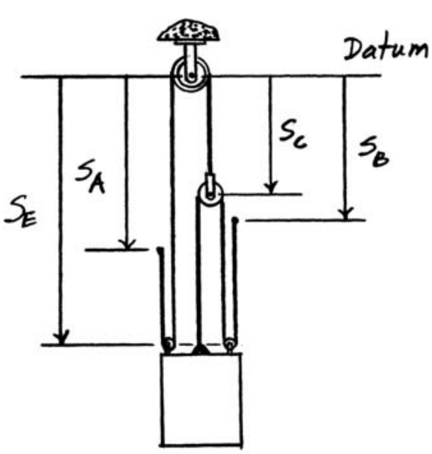
Time Derivative: Taking the time derivative of the above equation,

$$(+\downarrow) \qquad 7v_E - 2v_A - v_B = 0$$

Here, $v_A = v_B = -5$ m/s. Thus,

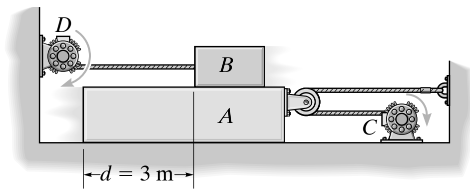
$$7v_E - \left[2(-5)\right] - (-5) = 0$$

$$v_E = -2.14\text{ m/s} = 2.14\text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(a)

12–210. The motor at C pulls in the cable with an acceleration $a_C = (3t^2)$ m/s^2, where t is in seconds. The motor at D draws in its cable at $a_D = 5$ m/s^2. If both motors start at the same instant from rest when $d = 3$ m, determine (a) the time needed for $d = 0$, and (b) the velocities of blocks A and B when this occurs.



For A:

$s_A + (s_A - s_C) = l$

$2v_A = v_C$

$2a_A = a_C = -3t^2$

$a_A = -1.5t^2 = 1.5t^2 \rightarrow$

$v_A = 0.5t^3 \rightarrow$

$s_A = 0.125t^4 \rightarrow$

For B:

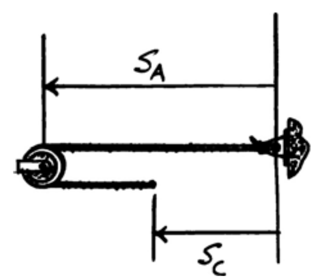
$a_B = 5 \text{ m/s}^2 \leftarrow$

$v_B = 5t \leftarrow$

$s_B = 2.5t^2 \leftarrow$

Require $s_A + s_B = d$

$0.125t^4 + 2.5t^2 = 3$



Set $u = t^2$ $\quad 0.125u^2 + 2.5u = 3$

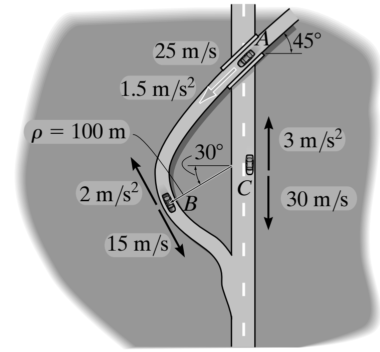
The positive root is $u = 1.1355$. Thus,

$$t = 1.0656 = 1.07 \text{ s}$$ **Ans.**

$$v_A = .0.5(1.0656)^2 = 0.605 \text{ m/s}$$ **Ans.**

$$v_B = 5(1.0656) = 5.33 \text{ m/s}$$ **Ans.**

•12–217. Car *B* is traveling along the curved road with a speed of 15 m/s while decreasing its speed at 2 m/s^2. At this same instant car *C* is traveling along the straight road with a speed of 30 m/s while decelerating at 3 m/s^2. Determine the velocity and acceleration of car *B* relative to car *C*.



Velocity: The velocity of cars *B* and *C* expressed in Cartesian vector form are

$$\mathbf{v}_B = [15\cos 60°\,\mathbf{i} - 15\sin 60°\,\mathbf{j}]\ \text{m/s} = [7.5\mathbf{i} - 12.99\mathbf{j}]\ \text{m/s}$$

$$\mathbf{v}_C = [-30\mathbf{j}]\ \text{m/s}$$

Applying the relative velocity equation,

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$7.5\mathbf{i} - 12.99\mathbf{j} = -30\mathbf{j} + \mathbf{v}_{B/C}$$

$$\mathbf{v}_{B/C} = [7.5\mathbf{i} + 17.01\mathbf{j}]\ \text{m/s}$$

Thus, the magnitude of $\mathbf{v}_{B/C}$ is given by

$$v_{BC} = \sqrt{7.5^2 + 17.01^2} = 18.6\ \text{m/s} \qquad \textbf{Ans.}$$

and the direction angle θ_v that $\mathbf{v}_{B/C}$ makes with the x axis is

$$\theta_v = \tan^{-1}\left(\frac{17.01}{7.5}\right) = 66.2° \qquad \textbf{Ans.}$$

Acceleration: The normal component of car *B*'s acceleration is $(a_B)_n = \dfrac{v_B{}^2}{\rho} = \dfrac{15^2}{100} = 2.25\ \text{m/s}^2$. Thus, the tangential and normal components of car *B*'s acceleration and the acceleration of car *C* expressed in Cartesian vector form are

$$(\mathbf{a}_B)_t = [-2\cos 60°\,\mathbf{i} + 2\sin 60°\mathbf{j}] = [-1\mathbf{i} + 1.732\mathbf{j}]\ \text{m/s}^2$$

$$(\mathbf{a}_B)_n = [2.25\cos 30°\,\mathbf{i} + 2.25\sin 30°\,\mathbf{j}] = [1.9486\mathbf{i} + 1.125\mathbf{j}]\ \text{m/s}^2$$

$$\mathbf{a}_C = [3\mathbf{j}]\ \text{m/s}^2$$

Applying the relative acceleration equation,

$$\mathbf{a}_B = \mathbf{a}_C + \mathbf{a}_{B/C}$$

$$(-1\mathbf{i} + 1.732\mathbf{j}) + (1.9486\mathbf{i} + 1.125\mathbf{j}) = 3\mathbf{j} + \mathbf{a}_{B/C}$$

$$\mathbf{a}_{B/C} = [0.9486\mathbf{i} - 0.1429\mathbf{j}]\ \text{m/s}^2$$

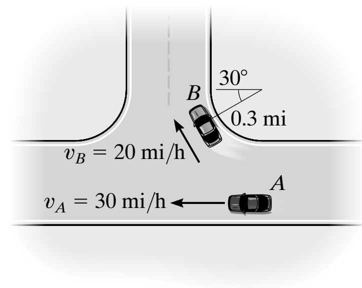
Thus, the magnitude of $\mathbf{a}_{B/C}$ is given by

$$a_{B/C} = \sqrt{0.9486^2 + (-0.1429)^2} = 0.959\ \text{m/s}^2 \qquad \textbf{Ans.}$$

and the direction angle θ_a that $\mathbf{a}_{B/C}$ makes with the x axis is

$$\theta_a = \tan^{-1}\left(\frac{0.1429}{0.9486}\right) = 8.57° \qquad \textbf{Ans.}$$

•12–221. At the instant shown, cars A and B travel at speeds of 30 mi/h and 20 mi/h, respectively. If B is increasing its speed by 1200 mi/h^2, while A maintains a constant speed, determine the velocity and acceleration of B with respect to A.



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\underset{\nwarrow 30^\circ}{20} = \underset{\leftarrow}{30} + \underset{\rightarrow}{(v_{B/A})_x} + \underset{\uparrow}{(v_{B/A})_y}$$

$(\overset{+}{\rightarrow})$ $\quad -20 \sin 30^\circ = -30 + (v_{B/A})_x$

$(+\uparrow)$ $\quad 20 \cos 30^\circ = (v_{B/A})_y$

Solving

$(v_{B/A})_x = 20 \rightarrow$

$(v_{B/A})_y = 17.32 \uparrow$

$v_{B/A} = \sqrt{(20)^2 + (17.32)^2} = 26.5 \text{ mi/h}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{17.32}{20}\right) = 40.9^\circ$ ⦣θ **Ans.**

$$(a_B)_n = \frac{(20)^2}{0.3} = 1333.3$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\underset{\nwarrow 30^\circ}{1200} + \underset{30^\circ}{\overset{1333.3}{\nearrow \theta}} = 0 + \underset{\rightarrow}{(a_{B/A})_x} + \underset{\uparrow}{(a_{B/A})_y}$$

$(\overset{+}{\rightarrow})$ $\quad -1200 \sin 30^\circ + 1333.3 \cos 30^\circ = (a_{B/A})_x$

$(+\uparrow)$ $\quad 1200 \cos 30^\circ + 1333.3 \sin 30^\circ = (a_{B/A})_y$

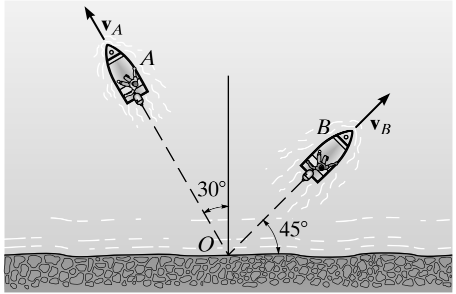
Solving

$(a_{B/A})_x = 554.7 \rightarrow \quad ; \quad (a_{B/A})_y = 1705.9 \uparrow$

$a_{B/A} = \sqrt{(554.7)^2 + 1705.9)^2} = 1.79(10^3) \text{ mi/h}^2$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{1705.9}{554.7}\right) = 72.0^\circ$ ⦣θ **Ans.**

12–223. Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 20\,\text{ft/s}$ and $v_B = 15\,\text{ft/s}$, determine the velocity of boat A with respect to boat B. How long after leaving the shore will the boats be 800 ft apart?



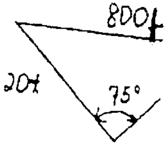
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-20 \sin 30°\mathbf{i} + 20 \cos 30°\mathbf{j} = 15 \cos 45°\mathbf{i} + 15 \sin 45°\mathbf{j} + \mathbf{v}_{A/B}$$

$$\mathbf{v}_{A/B} = \{-20.61\mathbf{i} + 6.714\mathbf{j}\}\ \text{ft/s}$$

$$v_{A/B} = \sqrt{(-20.61)^2 + (+6.714)^2} = 21.7\ \text{ft/s}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{6.714}{20.61}\right) = 18.0°$$ **Ans.**



$$(800)^2 = (20\,t)^2 + (15\,t)^2 - 2(20\,t)(15\,t) \cos 75°$$

$$t = 36.9\ \text{s}$$ **Ans.**

Also

$$t = \frac{800}{v_{A/B}} = \frac{800}{21.68} = 36.9\ \text{s}$$ **Ans.**