EXAMPLE 16.1

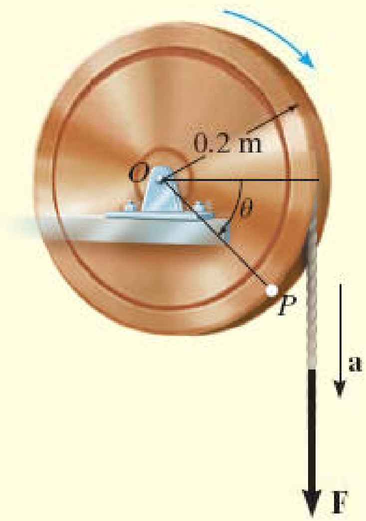


Fig. 16–5

A cord is wrapped around a wheel in Fig. 16–5, which is initially at rest when $\theta = 0$. If a force is applied to the cord and gives it an acceleration $a = (4t)\ \text{m/s}^2$, where t is in seconds, determine, as a function of time, (a) the angular velocity of the wheel, and (b) the angular position of line OP in radians.

SOLUTION

Part (a). The wheel is subjected to rotation about a fixed axis passing through point O. Thus, point P on the wheel has motion about a circular path, and the acceleration of this point has *both* tangential and normal components. The tangential component is $(a_P)_t = (4t)\ \text{m/s}^2$, since the cord is wrapped around the wheel and moves *tangent* to it. Hence the angular acceleration of the wheel is

$$(\circlearrowright +) \qquad (a_P)_t = \alpha r$$

$$(4t)\ \text{m/s}^2 = \alpha(0.2\ \text{m})$$

$$\alpha = (20t)\ \text{rad/s}^2 \circlearrowright$$

EXAMPLE 16.1 CONTINUED

Using this result, the wheel's angular velocity ω can now be determined from $\alpha = d\omega/dt$, since this equation relates α, t, and ω. Integrating, with the initial condition that $\omega = 0$ when $t = 0$, yields

$$(\circlearrowleft +) \qquad \alpha = \frac{d\omega}{dt} = (20t)\ \text{rad/s}^2$$

$$\int_0^{\omega} d\omega = \int_0^t 20t\, dt$$

$$\omega = 10t^2\ \text{rad/s} \circlearrowright \qquad \textit{Ans.}$$

Part (b). Using this result, the angular position θ of OP can be found from $\omega = d\theta/dt$, since this equation relates θ, ω, and t. Integrating, with the initial condition $\theta = 0$ when $t = 0$, we have

$$(\circlearrowleft +) \qquad \frac{d\theta}{dt} = \omega = (10t^2)\ \text{rad/s}$$

$$\int_0^{\theta} d\theta = \int_0^t 10t^2\, dt$$

$$\theta = 3.33t^3\ \text{rad} \qquad \textit{Ans.}$$

NOTE: We cannot use the equation of constant angular acceleration, since α is a function of time.

EXAMPLE 16.2

The motor shown in the photo is used to turn a wheel and attached blower contained within the housing. The details of the design are shown in Fig. 16–6*a*. If the pulley A connected to the motor begins to rotate from rest with a constant angular acceleration of $\alpha_A = 2\ \text{rad/s}^2$, determine the magnitudes of the velocity and acceleration of point P on the wheel, after the pulley has turned two revolutions. Assume the transmission belt does not slip on the pulley and wheel.

SOLUTION

Angular Motion. First we will convert the two revolutions to radians. Since there are 2π rad in one revolution, then

$$\theta_A = 2\ \text{rev}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 12.57\ \text{rad}$$

EXAMPLE 16.2 CONTINUED

Since α_A is constant, the angular velocity of pulley A is therefore

$$(\circlearrowright +) \qquad \omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega_A^2 = 0 + 2(2\ \text{rad/s}^2)(12.57\ \text{rad} - 0)$$

$$\omega_A = 7.090\ \text{rad/s}$$

The belt has the same speed and tangential component of acceleration as it passes over the pulley and wheel. Thus,

$$v = \omega_A r_A = \omega_B r_B; \; 7.090\ \text{rad/s}\ (0.15\ \text{m}) = \omega_B (0.4\ \text{m})$$

$$\omega_B = 2.659\ \text{rad/s}$$

$$a_t = \alpha_A r_A = \alpha_B r_B; \; 2\ \text{rad/s}^2\ (0.15\ \text{m}) = \alpha_B (0.4\ \text{m})$$

$$\alpha_B = 0.750\ \text{rad/s}^2$$

Motion of *P*. As shown on the kinematic diagram in Fig. 16–6*b*, we have

$$v_P = \omega_B r_B = 2.659\ \text{rad/s}\ (0.4\ \text{m}) = 1.06\ \text{m/s} \qquad \textit{Ans.}$$

$$(a_P)_t = \alpha_B r_B = 0.750\ \text{rad/s}^2\ (0.4\ \text{m}) = 0.3\ \text{m/s}^2$$

$$(a_P)_n = \omega_B^2 r_B = (2.659\ \text{rad/s})^2 (0.4\ \text{m}) = 2.827\ \text{m/s}^2$$

Thus

$$a_P = \sqrt{(0.3\ \text{m/s}^2)^2 + (2.827\ \text{m/s}^2)^2} = 2.84\ \text{m/s}^2 \qquad \textit{Ans.}$$

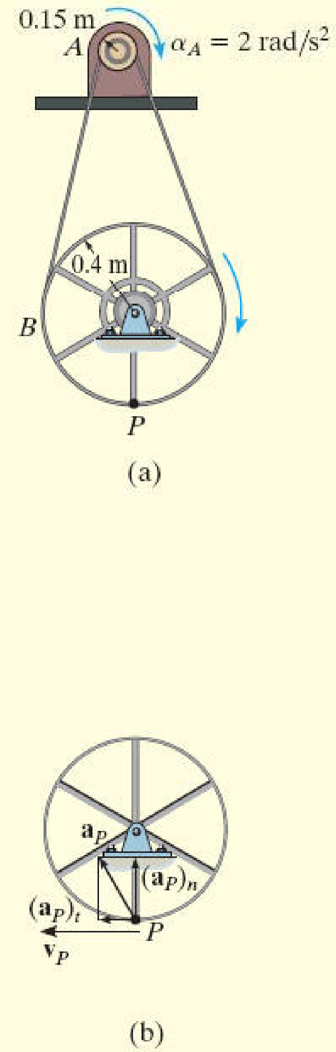


Fig. 16–6

EXAMPLE 16.3

The end of rod R shown in Fig. 16–7 maintains contact with the cam by means of a spring. If the cam rotates about an axis passing through point O with an angular acceleration $\boldsymbol{\alpha}$ and angular velocity $\boldsymbol{\omega}$, determine the velocity and acceleration of the rod when the cam is in the arbitrary position θ.

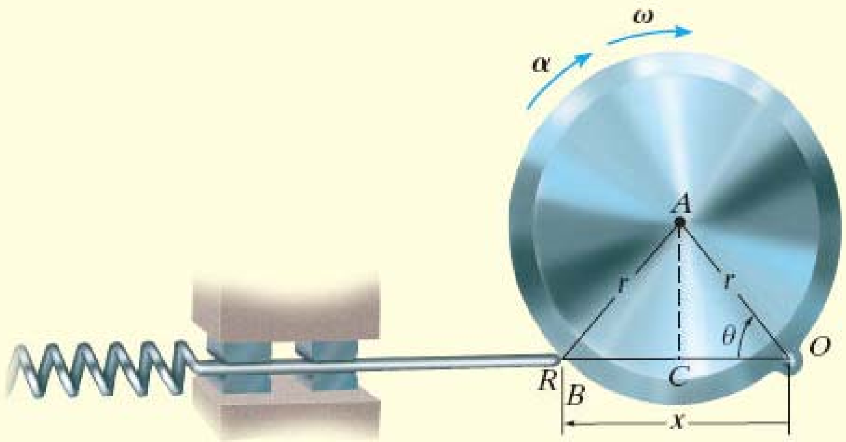


Fig. 16–7

EXAMPLE 16.3 CONTINUED

SOLUTION

Position Coordinate Equation. Coordinates θ and x are chosen in order to relate the *rotational motion* of the line segment OA on the cam to the *rectilinear translation* of the rod. These coordinates are measured from the *fixed point* O and can be related to each other using trigonometry. Since $OC = CB = r \cos \theta$, Fig. 16–7, then

$$x = 2r \cos \theta$$

Time Derivatives. Using the chain rule of calculus, we have

$$\frac{dx}{dt} = -2r(\sin \theta)\frac{d\theta}{dt}$$

$$v = -2r\omega \sin \theta \qquad \textit{Ans.}$$

$$\frac{dv}{dt} = -2r\left(\frac{d\omega}{dt}\right)\sin \theta - 2r\omega(\cos \theta)\frac{d\theta}{dt}$$

$$a = -2r(\alpha \sin \theta + \omega^2 \cos \theta) \qquad \textit{Ans.}$$

NOTE: The negative signs indicate that v and a are opposite to the direction of positive x. This seems reasonable when you visualize the motion.

EXAMPLE 16.4

At a given instant, the cylinder of radius r, shown in Fig. 16–8, has an angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$. Determine the velocity and acceleration of its center G if the cylinder rolls without slipping.

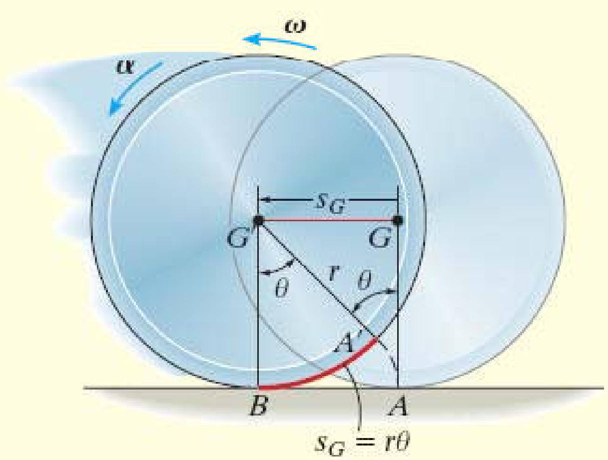


Fig. 16–8

EXAMPLE 16.4 CONTINUED

SOLUTION

Position Coordinate Equation. The cylinder undergoes general plane motion since it simultaneously translates and rotates. By inspection, point G moves in a *straight line* to the left, from G to G', as the cylinder rolls, Fig. 16–8. Consequently its new position G' will be specified by the *horizontal* position coordinate s_G, which is measured from G to G'. Also, as the cylinder rolls (without slipping), the arc length $A'B$ on the rim which was in contact with the ground from A to B, is equivalent to s_G. Consequently, the motion requires the radial line GA to rotate θ to the position $G'A'$. Since the arc $A'B = r\theta$, then G travels a distance

$$s_G = r\theta$$

Time Derivatives. Taking successive time derivatives of this equation, realizing that r is constant, $\omega = d\theta/dt$, and $\alpha = d\omega/dt$, gives the necessary relationships:

$$s_G = r\theta$$

$$v_G = r\omega \qquad \textit{Ans.}$$

$$a_G = r\alpha \qquad \textit{Ans.}$$

NOTE: Remember that these relationships are valid only if the cylinder (disk, wheel, ball, etc.) rolls *without* slipping.

EXAMPLE 16.5

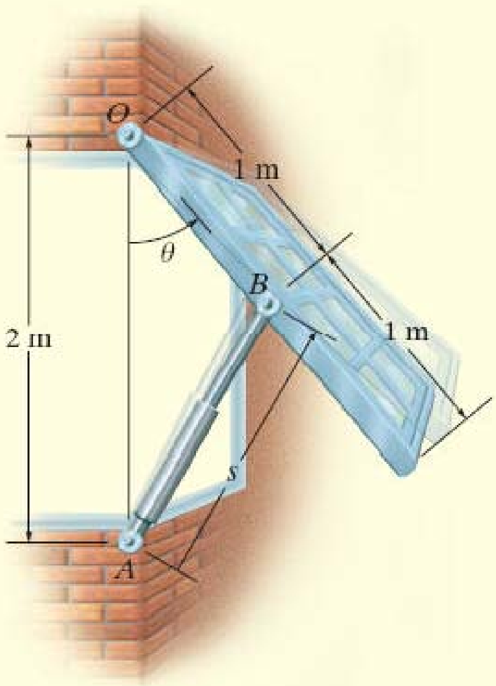


Fig. 16–9

The large window in Fig. 16–9 is opened using a hydraulic cylinder AB. If the cylinder extends at a constant rate of 0.5 m/s, determine the angular velocity and angular acceleration of the window at the instant $\theta = 30°$.

SOLUTION

Position Coordinate Equation. The angular motion of the window can be obtained using the coordinate θ, whereas the extension or motion *along the hydraulic cylinder* is defined using a coordinate s, which measures its length from the fixed point A to the moving point B. These coordinates can be related using the law of cosines, namely,

$$s^2 = (2\text{ m})^2 + (1\text{ m})^2 - 2(2\text{ m})(1\text{ m})\cos\theta$$

$$s^2 = 5 - 4\cos\theta \qquad (1)$$

When $\theta = 30°$,

$$s = 1.239\text{ m}$$

EXAMPLE 16.5 CONTINUED

Time Derivatives. Taking the time derivatives of Eq. 1, we have

$$2s\frac{ds}{dt} = 0 - 4(-\sin\theta)\frac{d\theta}{dt}$$

$$s(v_s) = 2(\sin\theta)\omega \qquad (2)$$

Since $v_s = 0.5$ m/s, then at $\theta = 30°$,

$$(1.239 \text{ m})(0.5 \text{ m/s}) = 2\sin 30°\omega$$

$$\omega = 0.6197 \text{ rad/s} = 0.620 \text{ rad/s} \qquad \textit{Ans.}$$

Taking the time derivative of Eq. 2 yields

$$\frac{ds}{dt}v_s + s\frac{dv_s}{dt} = 2(\cos\theta)\frac{d\theta}{dt}\omega + 2(\sin\theta)\frac{d\omega}{dt}$$

$$v_s^2 + sa_s = 2(\cos\theta)\omega^2 + 2(\sin\theta)\alpha$$

Since $a_s = dv_s/dt = 0$, then

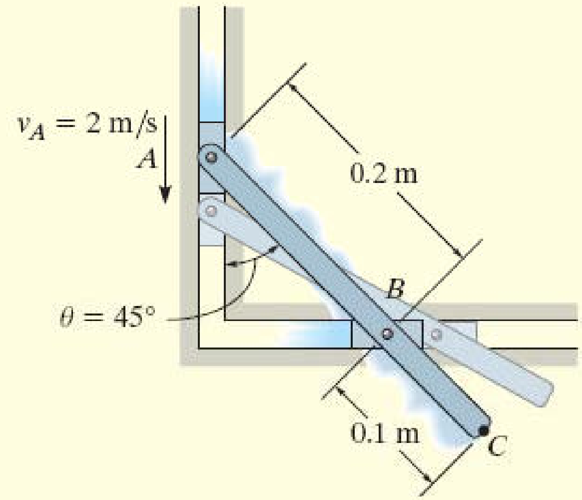
$$(0.5 \text{ m/s})^2 + 0 = 2\cos 30°(0.6197 \text{ rad/s})^2 + 2\sin 30°\alpha$$

$$\alpha = -0.415 \text{ rad/s}^2 \qquad \textit{Ans.}$$

Because the result is negative, it indicates the window has an angular deceleration.

EXAMPLE 16.6

The link shown in Fig. 16–13*a* is guided by two blocks at *A* and *B*, which move in the fixed slots. If the velocity of *A* is 2 m/s downward, determine the velocity of *B* at the instant $\theta = 45°$.



(a)

SOLUTION (VECTOR ANALYSIS)

Kinematic Diagram. Since points *A* and *B* are restricted to move along the fixed slots and $\mathbf{v}_A$ is directed downward, the velocity $\mathbf{v}_B$ must be directed horizontally to the right, Fig. 16–13*b*. This motion causes the link to rotate counterclockwise; that is, by the right-hand rule the angular velocity $\boldsymbol{\omega}$ is directed outward, perpendicular to the plane of motion. Knowing the magnitude and direction of $\mathbf{v}_A$ and the lines of action of $\mathbf{v}_B$ and $\boldsymbol{\omega}$, it is possible to apply the velocity equation $\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$ to points *A* and *B* in order to solve for the two unknown magnitudes v_B and ω. Since $\mathbf{r}_{B/A}$ is needed, it is also shown in Fig. 16–13*b*.

EXAMPLE 16.6 CONTINUED

Velocity Equation. Expressing each of the vectors in Fig. 16–13*b* in terms of their **i**, **j**, **k** components and applying Eq. 16–16 to *A*, the base point, and *B*, we have

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

$$v_B\mathbf{i} = -2\mathbf{j} + [\omega\mathbf{k} \times (0.2 \sin 45°\mathbf{i} - 0.2 \cos 45°\mathbf{j})]$$

$$v_B\mathbf{i} = -2\mathbf{j} + 0.2\omega \sin 45°\mathbf{j} + 0.2\omega \cos 45°\mathbf{i}$$

Equating the **i** and **j** components gives

$$v_B = 0.2\omega \cos 45° \qquad 0 = -2 + 0.2\omega \sin 45°$$

Thus,

$$\omega = 14.1 \text{ rad/s} \circlearrowleft$$

$$v_B = 2 \text{ m/s} \rightarrow \qquad \textit{Ans.}$$

Since both results are *positive*, the *directions* of $\mathbf{v}_B$ and $\boldsymbol{\omega}$ are indeed *correct* as shown in Fig. 16–13*b*. It should be emphasized that these results are *valid only* at the instant $\theta = 45°$. A recalculation for $\theta = 44°$ yields $v_B = 2.07$ m/s and $\omega = 14.4$ rad/s; whereas when $\theta = 46°$, $v_B = 1.93$ m/s and $\omega = 13.9$ rad/s, etc.

NOTE: Once the velocity of a point (*A*) on the link and the angular velocity are *known*, the velocity of any other point on the link can be determined. As an exercise, see if you can apply Eq. 16–16 to points *A* and *C* or to points *B* and *C* and show that when $\theta = 45°$, $v_C = 3.16$ m/s, directed at an angle of 18.4° up from the horizontal.

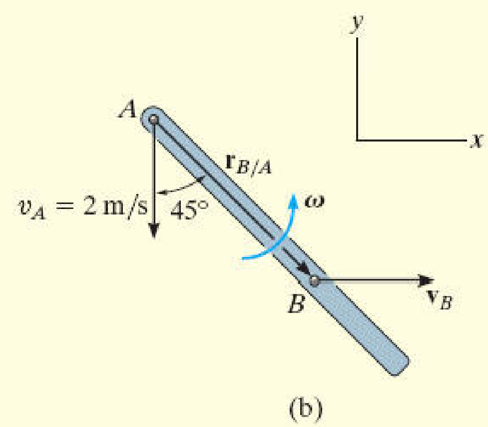
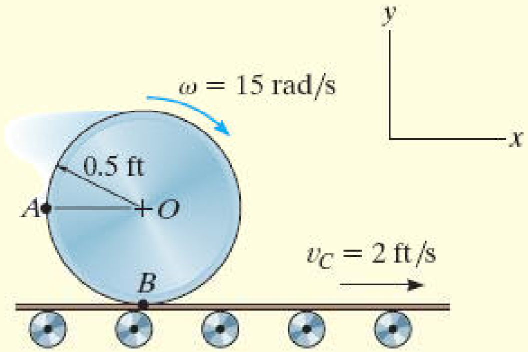
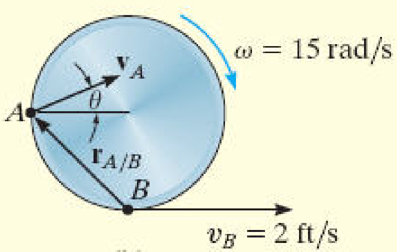


Fig. 16–13

EXAMPLE 16.7



(a)



(b)

The cylinder shown in Fig. 16–14*a* rolls without slipping on the surface of a conveyor belt which is moving at 2 ft/s. Determine the velocity of point A. The cylinder has a clockwise angular velocity $\omega = 15$ rad/s at the instant shown.

SOLUTION I (VECTOR ANALYSIS)

Kinematic Diagram. Since no slipping occurs, point B on the cylinder has the same velocity as the conveyor, Fig. 16–14*b*. Also, the angular velocity of the cylinder is known, so we can apply the velocity equation to B, the base point, and A to determine $\mathbf{v}_A$.

Velocity Equation.

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$(v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} = 2\mathbf{i} + (-15\mathbf{k}) \times (-0.5\mathbf{i} + 0.5\mathbf{j})$$

$$(v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} = 2\mathbf{i} + 7.50\mathbf{j} + 7.50\mathbf{i}$$

so that

$$(v_A)_x = 2 + 7.50 = 9.50 \text{ ft/s} \qquad (1)$$

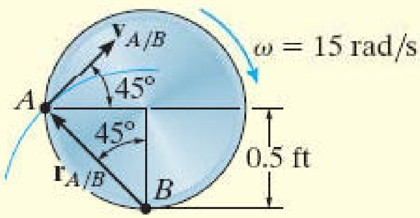
$$(v_A)_y = 7.50 \text{ ft/s} \qquad (2)$$

Thus,

$$v_A = \sqrt{(9.50)^2 + (7.50)^2} = 12.1 \text{ ft/s} \qquad \textit{Ans.}$$

$$\theta = \tan^{-1}\frac{7.50}{9.50} = 38.3^\circ \ \measuredangle \qquad \textit{Ans.}$$

EXAMPLE 16.7 CONTINUED



Relative motion
(c)

Fig. 16–14

SOLUTION II (SCALAR ANALYSIS)

As an alternative procedure, the scalar components of $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$ can be obtained directly. From the kinematic diagram showing the relative "circular" motion which produces $\mathbf{v}_{A/B}$, Fig. 16–14*c*, we have

$$v_{A/B} = \omega r_{A/B} = (15 \text{ rad/s})\left(\frac{0.5 \text{ ft}}{\cos 45^\circ}\right) = 10.6 \text{ ft/s}$$

Thus,

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$\begin{bmatrix}(v_A)_x \\ \rightarrow\end{bmatrix} + \begin{bmatrix}(v_A)_y \\ \uparrow\end{bmatrix} = \begin{bmatrix}2 \text{ ft/s} \\ \rightarrow\end{bmatrix} + \begin{bmatrix}10.6 \text{ ft/s} \\ \measuredangle\, 45^\circ\end{bmatrix}$$

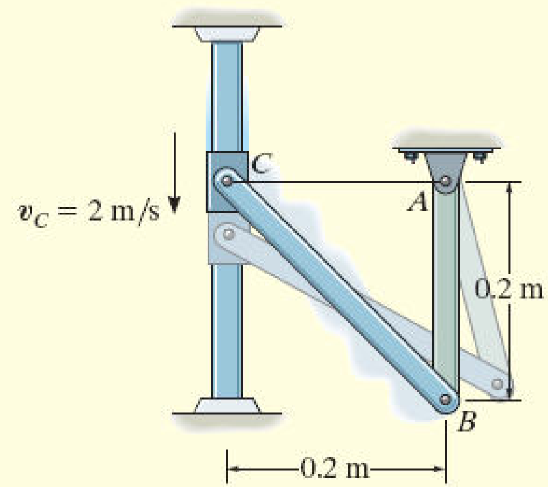
Equating the x and y components gives the same results as before, namely,

$$(\overset{+}{\rightarrow}) \qquad (v_A)_x = 2 + 10.6 \cos 45^\circ = 9.50 \text{ ft/s}$$

$$(+\uparrow) \qquad (v_A)_y = 0 + 10.6 \sin 45^\circ = 7.50 \text{ ft/s}$$

EXAMPLE 16.8

The collar C in Fig. 16–15a is moving downward with a velocity of 2 m/s. Determine the angular velocity of CB at this instant.



(a)

SOLUTION I (VECTOR ANALYSIS)

Kinematic Diagram. The downward motion of C causes B to move to the right along a curved path. Also, CB and AB rotate counterclockwise.

Velocity Equation. Link CB (general plane motion): See Fig. 16–15b.

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{CB} \times \mathbf{r}_{B/C}$$

$$v_B\mathbf{i} = -2\mathbf{j} + \omega_{CB}\mathbf{k} \times (0.2\mathbf{i} - 0.2\mathbf{j})$$

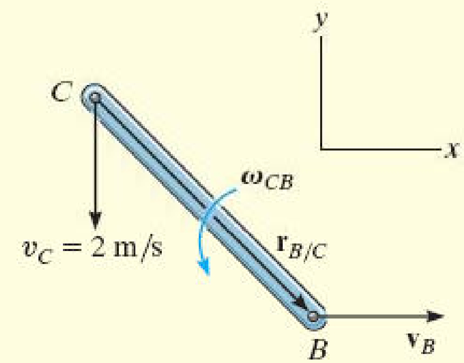
$$v_B\mathbf{i} = -2\mathbf{j} + 0.2\omega_{CB}\mathbf{j} + 0.2\omega_{CB}\mathbf{i}$$

$$v_B = 0.2\omega_{CB} \qquad (1)$$

$$0 = -2 + 0.2\omega_{CB} \qquad (2)$$

$$\omega_{CB} = 10 \text{ rad/s} \circlearrowleft \qquad \textit{Ans.}$$

$$v_B = 2 \text{ m/s} \rightarrow$$



(b)

EXAMPLE 16.8 CONTINUED

SOLUTION II (SCALAR ANALYSIS)

The scalar component equations of $\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$ can be obtained directly. The kinematic diagram in Fig. 16–15*c* shows the relative "circular" motion which produces $\mathbf{v}_{B/C}$. We have

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$\begin{bmatrix} v_B \\ \rightarrow \end{bmatrix} = \begin{bmatrix} 2\text{ m/s} \\ \downarrow \end{bmatrix} + \begin{bmatrix} \omega_{CB}\left(0.2\sqrt{2}\text{ m}\right) \\ \measuredangle 45^\circ \end{bmatrix}$$

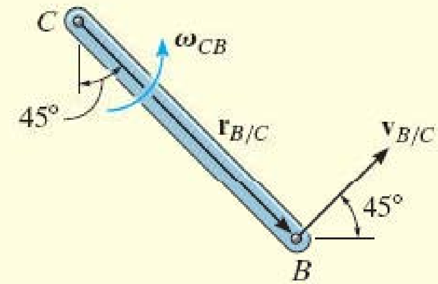
Resolving these vectors in the x and y directions yields

$$(\overset{+}{\rightarrow}) \qquad v_B = 0 + \omega_{CB}\left(0.2\sqrt{2}\cos 45^\circ\right)$$

$$(+\uparrow) \qquad 0 = -2 + \omega_{CB}\left(0.2\sqrt{2}\sin 45^\circ\right)$$

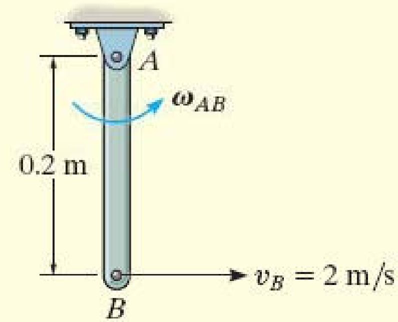
which is the same as Eqs. 1 and 2.

NOTE: Since link AB rotates about a fixed axis and v_B is known, Fig. 16–15*d*, its angular velocity is found from $v_B = \omega_{AB} r_{AB}$ or $2\text{ m/s} = \omega_{AB}\,(0.2\text{ m})$, $\omega_{AB} = 10\text{ rad/s}$.



Relative motion

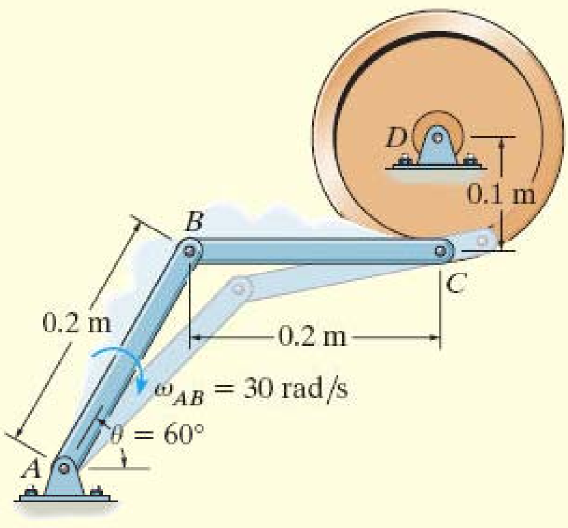
(c)



(d)

Fig. 16–15

EXAMPLE 16.9



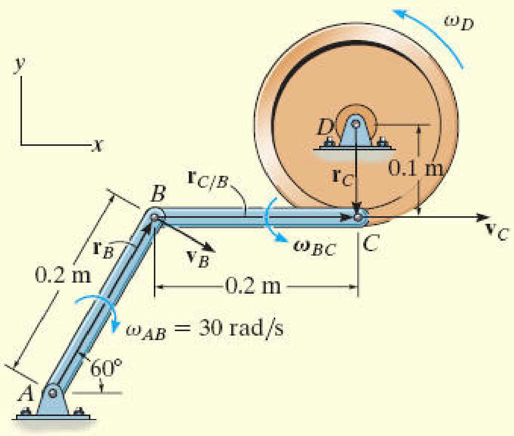
(a)

The bar AB of the linkage shown in Fig. 16–16a has a clockwise angular velocity of 30 rad/s when $\theta = 60°$. Determine the angular velocities of member BC and the wheel at this instant.

SOLUTION (VECTOR ANALYSIS)

Kinematic Diagram. By inspection, the velocities of points B and C are defined by the rotation of link AB and the wheel about their fixed axes. The position vectors and the angular velocity of each member are shown on the kinematic diagram in Fig. 16–16b. To solve, we will write the appropriate kinematic equation for each member.

EXAMPLE 16.9 CONTINUED



(b)

Fig. 16–16

Velocity Equation. Link AB (rotation about a fixed axis):

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_B$$
$$= (-30\mathbf{k}) \times (0.2 \cos 60°\mathbf{i} + 0.2 \sin 60°\mathbf{j})$$
$$= \{5.20\mathbf{i} - 3.0\mathbf{j}\} \text{ m/s}$$

Link BC (general plane motion):

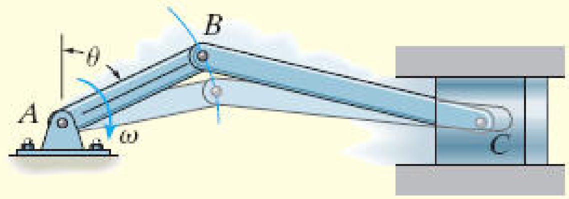
$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$
$$v_C\mathbf{i} = 5.20\mathbf{i} - 3.0\mathbf{j} + (\omega_{BC}\mathbf{k}) \times (0.2\mathbf{i})$$
$$v_C\mathbf{i} = 5.20\mathbf{i} + (0.2\omega_{BC} - 3.0)\mathbf{j}$$
$$v_C = 5.20 \text{ m/s}$$
$$0 = 0.2\omega_{BC} - 3.0$$
$$\omega_{BC} = 15 \text{ rad/s} \circlearrowleft \qquad \textit{Ans.}$$

Wheel (rotation about a fixed axis):

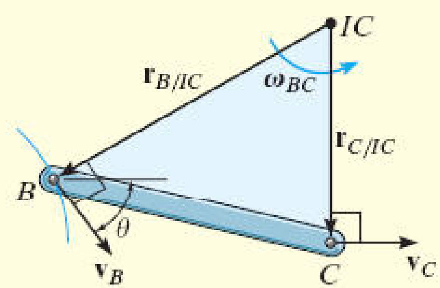
$$\mathbf{v}_C = \boldsymbol{\omega}_D \times \mathbf{r}_C$$
$$5.20\mathbf{i} = (\omega_D\mathbf{k}) \times (-0.1\mathbf{j})$$
$$5.20 = 0.1\omega_D$$
$$\omega_D = 52.0 \text{ rad/s} \circlearrowleft \qquad \textit{Ans.}$$

EXAMPLE 16.10

Show how to determine the location of the instantaneous center of zero velocity for (a) member *BC* shown in Fig. 16–20*a*; and (b) the link *CB* shown in Fig. 16–20*c*.



(a)



(b)

EXAMPLE 16.10 CONTINUED

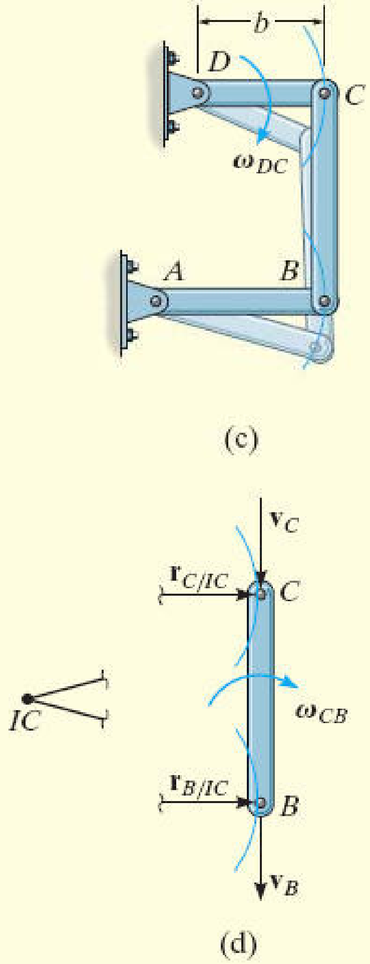


Fig. 16–20

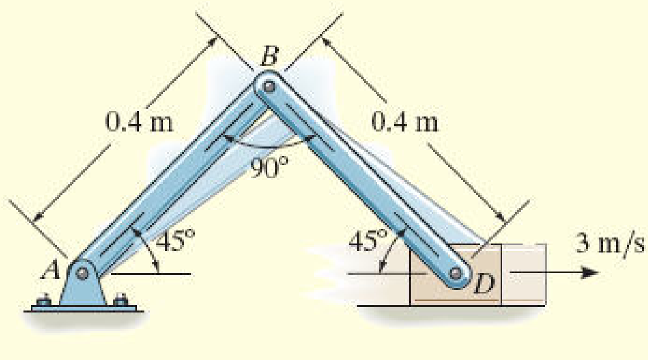
SOLUTION

Part (a). As shown in Fig. 16–20*a*, point *B* moves in a circular path such that $\mathbf{v}_B$ is perpendicular to *AB*. Therefore, it acts at an angle θ from the horizontal as shown in Fig. 16–20*b*. The motion of point *B* causes the piston to move forward *horizontally* with a velocity $\mathbf{v}_C$. When lines are drawn perpendicular to $\mathbf{v}_B$ and $\mathbf{v}_C$, Fig. 16–20*b*, they intersect at the *IC*.

Part (b). Points *B* and *C* follow circular paths of motion since links *AB* and *DC* are each subjected to rotation about a fixed axis, Fig. 16–20*c*. Since the velocity is always tangent to the path, at the instant considered, $\mathbf{v}_C$ on rod *DC* and $\mathbf{v}_B$ on rod *AB* are both directed vertically downward, along the axis of link *CB*, Fig. 16–20*d*. Radial lines drawn perpendicular to these two velocities form parallel lines which intersect at “infinity;” i.e., $r_{C/IC} \to \infty$ and $r_{B/IC} \to \infty$. Thus, $\omega_{CB} = (v_C/r_{C/IC}) \to 0$. As a result, link *CB* momentarily *translates*. An instant later, however, *CB* will move to a tilted position, causing the *IC* to move to some finite location.

EXAMPLE 16.11

Block D shown in Fig. 16–21a moves with a speed of 3 m/s. Determine the angular velocities of links BD and AB, at the instant shown.



(a)

EXAMPLE 16.11 CONTINUED

SOLUTION

As D moves to the right, it causes AB to rotate clockwise about point A. Hence, $\mathbf{v}_B$ is directed perpendicular to AB. The instantaneous center of zero velocity for BD is located at the intersection of the line segments drawn perpendicular to $\mathbf{v}_B$ and $\mathbf{v}_D$, Fig. 16–21b. From the geometry,

$$r_{B/IC} = 0.4 \tan 45^\circ \text{ m} = 0.4 \text{ m}$$

$$r_{D/IC} = \frac{0.4 \text{ m}}{\cos 45^\circ} = 0.5657 \text{ m}$$

Since the magnitude of $\mathbf{v}_D$ is known, the angular velocity of link BD is

$$\omega_{BD} = \frac{v_D}{r_{D/IC}} = \frac{3 \text{ m/s}}{0.5657 \text{ m}} = 5.30 \text{ rad/s} \circlearrowleft \qquad \textit{Ans.}$$

The velocity of B is therefore

$$v_B = \omega_{BD}(r_{B/IC}) = 5.30 \text{ rad/s } (0.4 \text{ m}) = 2.12 \text{ m/s} \quad \measuredangle 45^\circ$$

From Fig. 16–21c, the angular velocity of AB is

$$\omega_{AB} = \frac{v_B}{r_{B/A}} = \frac{2.12 \text{ m/s}}{0.4 \text{ m}} = 5.30 \text{ rad/s} \circlearrowright \qquad \textit{Ans.}$$

NOTE: Try and solve this problem by applying $\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$ to member BD.

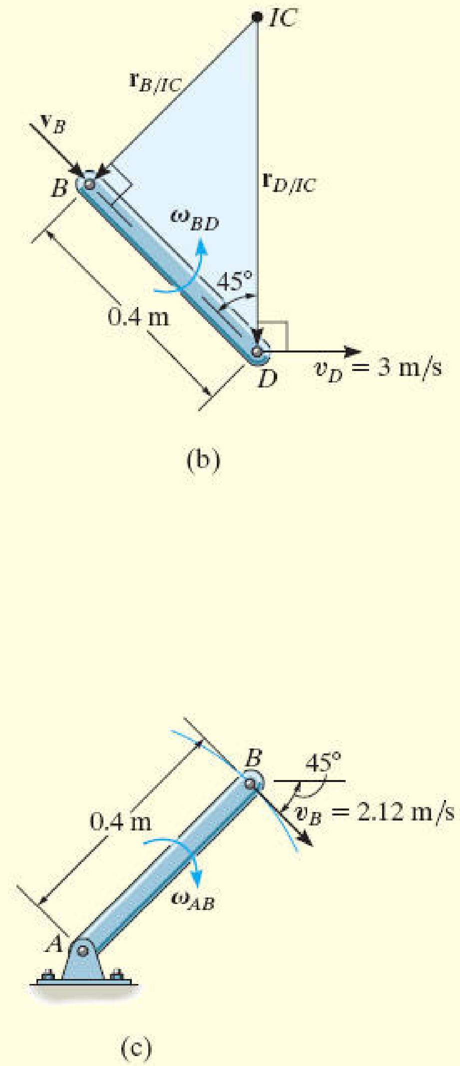
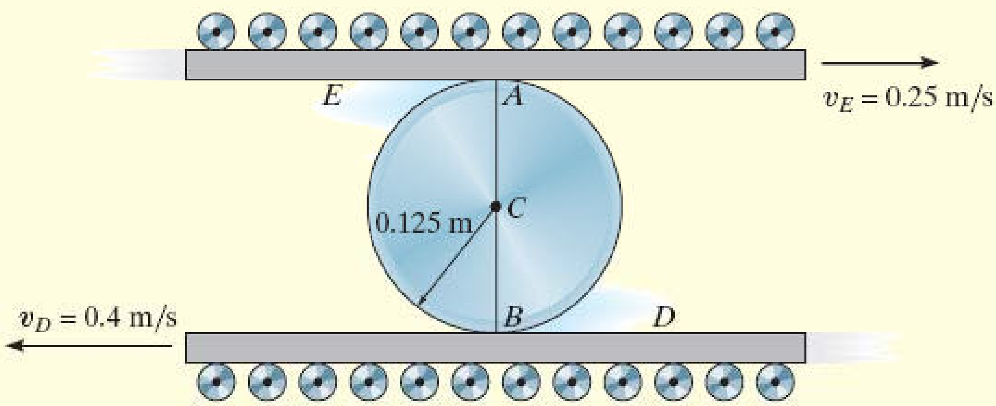


Fig. 16–21

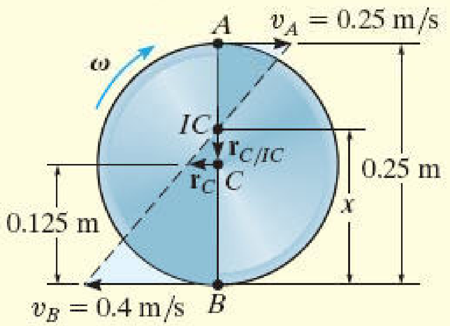
EXAMPLE 16.12

The cylinder shown in Fig. 16–22*a* rolls without slipping between the two moving plates *E* and *D*. Determine the angular velocity of the cylinder and the velocity of its center *C*.



(a)

EXAMPLE 16.12 CONTINUED



(b)

Fig. 16–22

SOLUTION

Since no slipping occurs, the contact points A and B on the cylinder have the same velocities as the plates E and D, respectively. Furthermore, the velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ are *parallel*, so that by the proportionality of right triangles the IC is located at a point on line AB, Fig. 16–22b. Assuming this point to be a distance x from B, we have

$$v_B = \omega x; \qquad 0.4 \text{ m/s} = \omega x$$

$$v_A = \omega(0.25 \text{ m} - x); \qquad 0.25 \text{ m/s} = \omega(0.25 \text{ m} - x)$$

Dividing one equation into the other eliminates ω and yields

$$0.4(0.25 - x) = 0.25x$$

$$x = \frac{0.1}{0.65} = 0.1538 \text{ m}$$

Hence, the angular velocity of the cylinder is

$$\omega = \frac{v_B}{x} = \frac{0.4 \text{ m/s}}{0.1538 \text{ m}} = 2.60 \text{ rad/s} \circlearrowright \qquad \textit{Ans.}$$

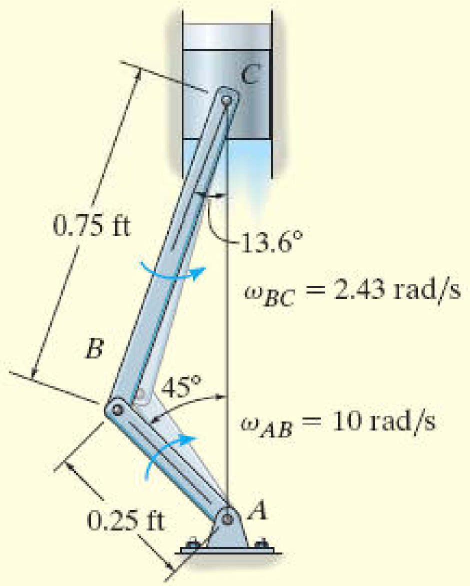
The velocity of point C is therefore

$$v_C = \omega r_{C/IC} = 2.60 \text{ rad/s}\,(0.1538 \text{ m} - 0.125 \text{ m})$$

$$= 0.0750 \text{ m/s} \leftarrow \qquad \textit{Ans.}$$

EXAMPLE 16.13

The crankshaft AB turns with a clockwise angular velocity of 10 rad/s, Fig. 16–23a. Determine the velocity of the piston at the instant shown.



(a)

EXAMPLE 16.13 CONTINUED

SOLUTION

The crankshaft rotates about a fixed axis, and so the velocity of point B is

$$v_B = 10 \text{ rad/s } (0.25 \text{ ft}) = 2.50 \text{ ft/s} \measuredangle 45^\circ$$

Since the directions of the velocities of B and C are known, then the location of the IC for the connecting rod BC is at the intersection of the lines extended from these points, perpendicular to $\mathbf{v}_B$ and $\mathbf{v}_C$, Fig. 16–23b. The magnitudes of $\mathbf{r}_{B/IC}$ and $\mathbf{r}_{C/IC}$ can be obtained from the geometry of the triangle and the law of sines, i.e.,

$$\frac{0.75 \text{ ft}}{\sin 45^\circ} = \frac{r_{B/IC}}{\sin 76.4^\circ}$$

$$r_{B/IC} = 1.031 \text{ ft}$$

$$\frac{0.75 \text{ ft}}{\sin 45^\circ} = \frac{r_{C/IC}}{\sin 58.6^\circ}$$

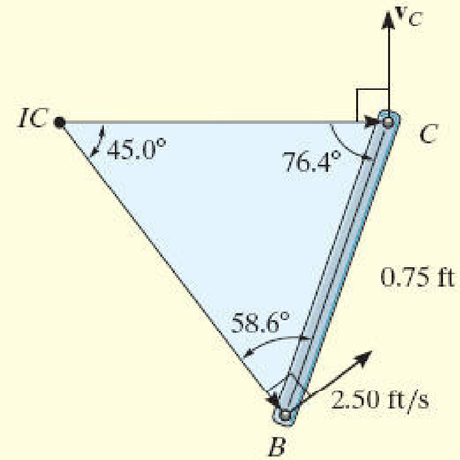
$$r_{C/IC} = 0.9056 \text{ ft}$$

The rotational sense of $\boldsymbol{\omega}_{BC}$ must be the same as the rotation caused by $\mathbf{v}_B$ about the IC, which is counterclockwise. Therefore,

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{2.5 \text{ ft/s}}{1.031 \text{ ft}} = 2.425 \text{ rad/s}$$

Using this result, the velocity of the piston is

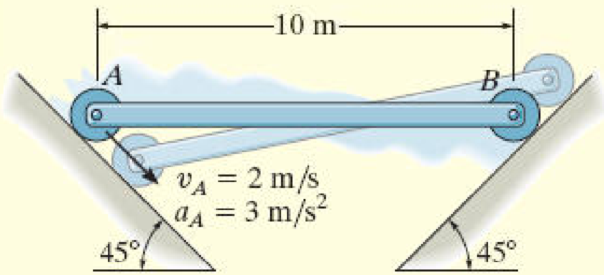
$$v_C = \omega_{BC} r_{C/IC} = (2.425 \text{ rad/s})(0.9056 \text{ ft}) = 2.20 \text{ ft/s} \quad \textit{Ans.}$$



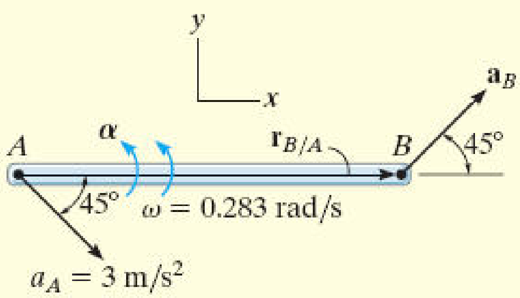
(b)

Fig. 16–23

EXAMPLE 16.14



(a)



(b)

The rod AB shown in Fig. 16–27a is confined to move along the inclined planes at A and B. If point A has an acceleration of 3 m/s^2 and a velocity of 2 m/s, both directed down the plane at the instant the rod is horizontal, determine the angular acceleration of the rod at this instant.

SOLUTION I (VECTOR ANALYSIS)

We will apply the acceleration equation to points A and B on the rod. To do so it is first necessary to determine the angular velocity of the rod. Show that it is $\omega = 0.283$ rad/s ↺ using either the velocity equation or the method of instantaneous centers.

Kinematic Diagram. Since points A and B both move along straight-line paths, they have *no* components of acceleration normal to the paths. There are two unknowns in Fig. 16–27b, namely, $\mathbf{a}_B$ and α.

EXAMPLE 16.14 CONTINUED

Acceleration Equation.

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$a_B \cos 45^\circ \mathbf{i} + a_B \sin 45^\circ \mathbf{j} = 3 \cos 45^\circ \mathbf{i} - 3 \sin 45^\circ \mathbf{j} + (\alpha \mathbf{k}) \times (10\mathbf{i}) - (0.283)^2(10\mathbf{i})$$

Carrying out the cross product and equating the **i** and **j** components yields

$$a_B \cos 45^\circ = 3 \cos 45^\circ - (0.283)^2(10) \quad (1)$$

$$a_B \sin 45^\circ = -3 \sin 45^\circ + \alpha(10) \quad (2)$$

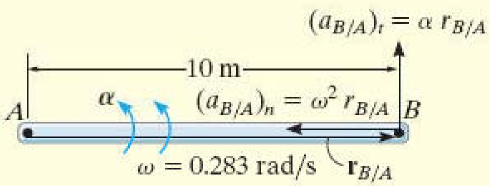
Solving, we have

$$a_B = 1.87 \text{ m/s}^2 \measuredangle 45^\circ$$

$$\alpha = 0.344 \text{ rad/s}^2 \circlearrowleft \qquad \textit{Ans.}$$

SOLUTION II (SCALAR ANALYSIS)

From the kinematic diagram, showing the relative-acceleration components $(\mathbf{a}_{B/A})_t$ and $(\mathbf{a}_{B/A})_n$, Fig. 16–27c, we have



(c)

Fig. 16–27

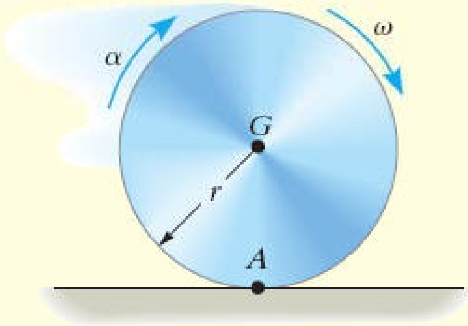
$$\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$$

$$\begin{bmatrix} a_B \\ \measuredangle 45^\circ \end{bmatrix} = \begin{bmatrix} 3 \text{ m/s}^2 \\ \triangleleft 45^\circ \end{bmatrix} + \begin{bmatrix} \alpha(10 \text{ m}) \\ \uparrow \end{bmatrix} + \begin{bmatrix} (0.283 \text{ rad/s})^2(10 \text{ m}) \\ \leftarrow \end{bmatrix}$$

Equating the x and y components yields Eqs. 1 and 2, and the solution proceeds as before.

EXAMPLE 16.15

At a given instant, the cylinder of radius r, shown in Fig. 16–28a, has an angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$. Determine the velocity and acceleration of its center G and the acceleration of the contact point at A if it rolls without slipping.



(a)

SOLUTION (VECTOR ANALYSIS)

Velocity Analysis. Since no slipping occurs, at the instant A contacts the ground, $\mathbf{v}_A = \mathbf{0}$. Thus, from the kinematic diagram in Fig. 16–28b we have

$$\mathbf{v}_G = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{G/A}$$

$$v_G\mathbf{i} = \mathbf{0} + (-\omega\mathbf{k}) \times (r\mathbf{j})$$

$$v_G = \omega r \qquad (1) \quad \textit{Ans.}$$

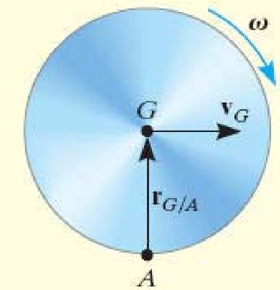
This same result can also be obtained directly by noting that point A represents the instantaneous center of zero velocity.

EXAMPLE 16.15 CONTINUED

Kinematic Diagram. Since the motion of G is always along a *straight line*, then its acceleration can be determined by taking the time derivative of its velocity, which gives

$$a_G = \frac{dv_G}{dt} = \frac{d\omega}{dt} r$$

$$a_G = \alpha r \qquad (2) \quad \textit{Ans.}$$



(b)

Acceleration Equation. The magnitude and direction of $\mathbf{a}_A$ is unknown, Fig. 16–28*c*.

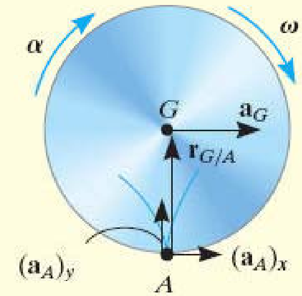
$$\mathbf{a}_G = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{G/A} - \omega^2 \mathbf{r}_{G/A}$$

$$\alpha r\,\mathbf{i} = (a_A)_x\mathbf{i} + (a_A)_y\mathbf{j} + (-\alpha\mathbf{k}) \times (r\mathbf{j}) - \omega^2(r\mathbf{j})$$

Evaluating the cross product and equating the **i** and **j** components yields

$$(a_A)_x = 0 \qquad \textit{Ans.}$$

$$(a_A)_y = \omega^2 r \qquad \textit{Ans.}$$

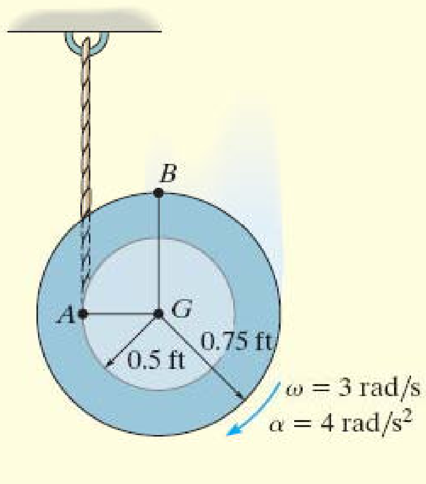


(c)

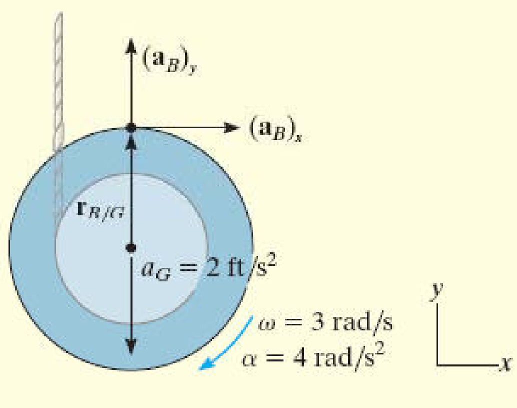
Fig. 16–28

NOTE: The results, that $v_G = \omega r$ and $a_G = \alpha r$, can be applied to any circular object, such as a ball, cylinder, disk, etc., that rolls *without* slipping. Also, the fact that $a_A = \omega^2 r$ indicates that the instantaneous center of zero velocity, point A, is *not* a point of zero acceleration.

EXAMPLE 16.16



(a)



(b)

The spool shown in Fig. 16–29*a* unravels from the cord, such that at the instant shown it has an angular velocity of 3 rad/s and an angular acceleration of 4 rad/s^2. Determine the acceleration of point *B*.

SOLUTION I (VECTOR ANALYSIS)

The spool "appears" to be rolling downward without slipping at point *A*. Therefore, we can use the results of Example 16.15 to determine the acceleration of point *G*, i.e.,

$$a_G = \alpha r = (4 \text{ rad/s}^2)(0.5 \text{ ft}) = 2 \text{ ft/s}^2$$

We will apply the acceleration equation to points *G* and *B*.

Kinematic Diagram. Point *B* moves along a *curved path* having an *unknown* radius of curvature.* Its acceleration will be represented by its unknown *x* and *y* components as shown in Fig. 16–29*b*.

Acceleration Equation.

$$\mathbf{a}_B = \mathbf{a}_G + \boldsymbol{\alpha} \times \mathbf{r}_{B/G} - \omega^2 \mathbf{r}_{B/G}$$

$$(a_B)_x\mathbf{i} + (a_B)_y\mathbf{j} = -2\mathbf{j} + (-4\mathbf{k}) \times (0.75\mathbf{j}) - (3)^2(0.75\mathbf{j})$$

Equating the **i** and **j** terms, the component equations are

$$(a_B)_x = 4(0.75) = 3 \text{ ft/s}^2 \rightarrow \qquad (1)$$

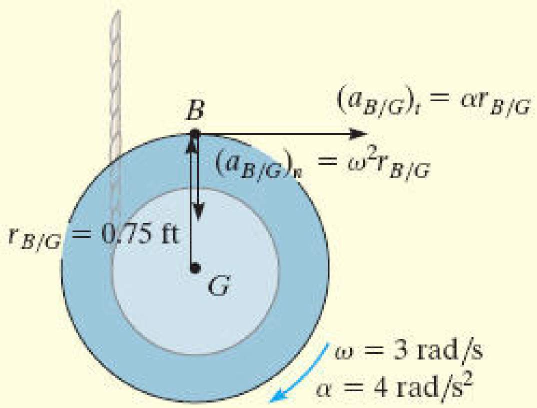
$$(a_B)_y = -2 - 6.75 = -8.75 \text{ ft/s}^2 = 8.75 \text{ ft/s}^2 \downarrow \qquad (2)$$

The magnitude and direction of $\mathbf{a}_B$ are therefore

$$a_B = \sqrt{(3)^2 + (8.75)^2} = 9.25 \text{ ft/s}^2 \qquad \textit{Ans.}$$

$$\theta = \tan^{-1}\frac{8.75}{3} = 71.1^\circ \qquad \textit{Ans.}$$

EXAMPLE 16.16 CONTINUED



(c)

Fig. 16–29

SOLUTION II (SCALAR ANALYSIS)

This problem may be solved by writing the scalar component equations directly. The kinematic diagram in Fig. 16–29*c* shows the relative-acceleration components $(\mathbf{a}_{B/G})_t$ and $(\mathbf{a}_{B/G})_n$. Thus,

$$\mathbf{a}_B = \mathbf{a}_G + (\mathbf{a}_{B/G})_t + (\mathbf{a}_{B/G})_n$$

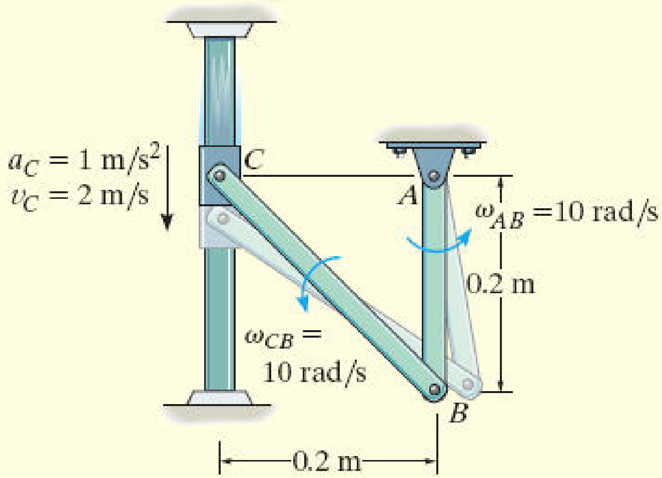
$$\begin{bmatrix} (a_B)_x \\ \rightarrow \end{bmatrix} + \begin{bmatrix} (a_B)_y \\ \uparrow \end{bmatrix} = \begin{bmatrix} 2 \text{ ft/s}^2 \\ \downarrow \end{bmatrix} + \begin{bmatrix} 4 \text{ rad/s}^2 (0.75 \text{ ft}) \\ \rightarrow \end{bmatrix} + \begin{bmatrix} (3 \text{ rad/s})^2 (0.75 \text{ ft}) \\ \downarrow \end{bmatrix}$$

The x and y components yield Eqs. 1 and 2 above.

*Realize that the path's radius of curvature ρ is not equal to the radius of the spool since the spool is not rotating about point G. Furthermore, ρ is not defined as the distance from A (IC) to B, since the location of the IC depends only on the velocity of a point and not the geometry of its path.

EXAMPLE 16.17

The collar C in Fig. 16–30a moves downward with an acceleration of $1\ \text{m/s}^2$. At the instant shown, it has a speed of 2 m/s which gives links CB and AB an angular velocity $\omega_{AB} = \omega_{CB} = 10$ rad/s. (See Example 16.8.) Determine the angular accelerations of CB and AB at this instant.



(a)

EXAMPLE 16.17 CONTINUED

SOLUTION (VECTOR ANALYSIS)

Kinematic Diagram. The kinematic diagrams of *both* links AB and CB are shown in Fig. 16–30*b*. To solve, we will apply the appropriate kinematic equation to each link.

Acceleration Equation.

Link AB (rotation about a fixed axis):

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_B - \omega_{AB}^2 \mathbf{r}_B$$

$$\mathbf{a}_B = (\alpha_{AB}\mathbf{k}) \times (-0.2\mathbf{j}) - (10)^2(-0.2\mathbf{j})$$

$$\mathbf{a}_B = 0.2\alpha_{AB}\mathbf{i} + 20\mathbf{j}$$

Note that $\mathbf{a}_B$ has n and t components since it moves along a *circular path*.

Link BC (general plane motion): Using the result for $\mathbf{a}_B$ and applying Eq. 16–18, we have

$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha}_{CB} \times \mathbf{r}_{B/C} - \omega_{CB}^2 \mathbf{r}_{B/C}$$

$$0.2\alpha_{AB}\mathbf{i} + 20\mathbf{j} = -1\mathbf{j} + (\alpha_{CB}\mathbf{k}) \times (0.2\mathbf{i} - 0.2\mathbf{j}) - (10)^2(0.2\mathbf{i} - 0.2\mathbf{j})$$

$$0.2\alpha_{AB}\mathbf{i} + 20\mathbf{j} = -1\mathbf{j} + 0.2\alpha_{CB}\mathbf{j} + 0.2\alpha_{CB}\mathbf{i} - 20\mathbf{i} + 20\mathbf{j}$$

Thus,

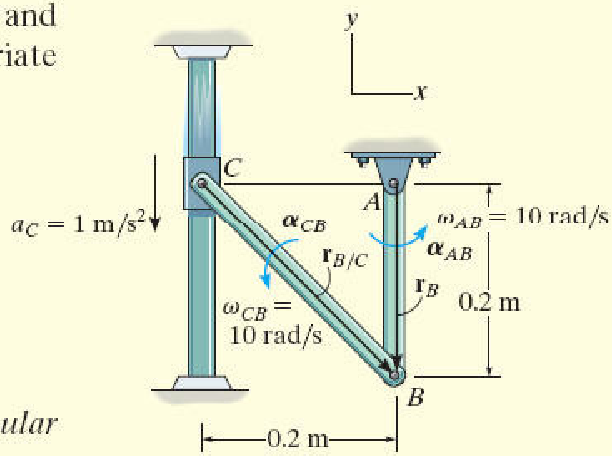
$$0.2\alpha_{AB} = 0.2\alpha_{CB} - 20$$

$$20 = -1 + 0.2\alpha_{CB} + 20$$

Solving,

$$\alpha_{CB} = 5 \text{ rad/s}^2 \circlearrowleft \qquad \textit{Ans.}$$

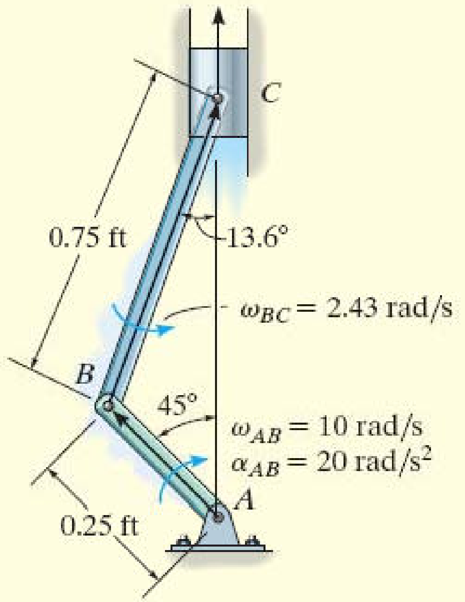
$$\alpha_{AB} = -95 \text{ rad/s}^2 = 95 \text{ rad/s}^2 \circlearrowright \qquad \textit{Ans.}$$



(b)

Fig. 16–30

EXAMPLE 16.18



(a)

The crankshaft AB turns with a clockwise angular acceleration of 20 rad/s^2, Fig. 16–31a. Determine the acceleration of the piston at the instant AB is in the position shown. At this instant $\omega_{AB} = 10$ rad/s and $\omega_{BC} = 2.43$ rad/s (See Example 16.13.)

SOLUTION (VECTOR ANALYSIS)

Kinematic Diagram. The kinematic diagrams for both AB and BC are shown in Fig. 16–31b. Here $\mathbf{a}_C$ is vertical since C moves along a straight-line path.

Acceleration Equation. Expressing each of the position vectors in Cartesian vector form

$$\mathbf{r}_B = \{-0.25 \sin 45°\mathbf{i} + 0.25 \cos 45°\mathbf{j}\} \text{ ft} = \{-0.177\mathbf{i} + 0.177\mathbf{j}\} \text{ ft}$$

$$\mathbf{r}_{C/B} = \{0.75 \sin 13.6°\mathbf{i} + 0.75 \cos 13.6°\mathbf{j}\} \text{ ft} = \{0.177\mathbf{i} + 0.729\mathbf{j}\} \text{ ft}$$

EXAMPLE 16.18 CONTINUED

Crankshaft *AB* (rotation about a fixed axis):

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_B - \omega_{AB}^2 \mathbf{r}_B$$

$$= (-20\mathbf{k}) \times (-0.177\mathbf{i} + 0.177\mathbf{j}) - (10)^2(-0.177\mathbf{i} + 0.177\mathbf{j})$$

$$= \{21.21\mathbf{i} - 14.14\mathbf{j}\}\ \text{ft/s}^2$$

Connecting Rod *BC* (general plane motion): Using the result for $\mathbf{a}_B$ and noting that $\mathbf{a}_C$ is in the vertical direction, we have

$$\mathbf{a}_C = \mathbf{a}_B + \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$$

$$a_C\mathbf{j} = 21.21\mathbf{i} - 14.14\mathbf{j} + (\alpha_{BC}\mathbf{k}) \times (0.177\mathbf{i} + 0.729\mathbf{j}) - (2.43)^2(0.177\mathbf{i} + 0.729\mathbf{j})$$

$$a_C\mathbf{j} = 21.21\mathbf{i} - 14.14\mathbf{j} + 0.177\alpha_{BC}\mathbf{j} - 0.729\alpha_{BC}\mathbf{i} - 1.04\mathbf{i} - 4.30\mathbf{j}$$

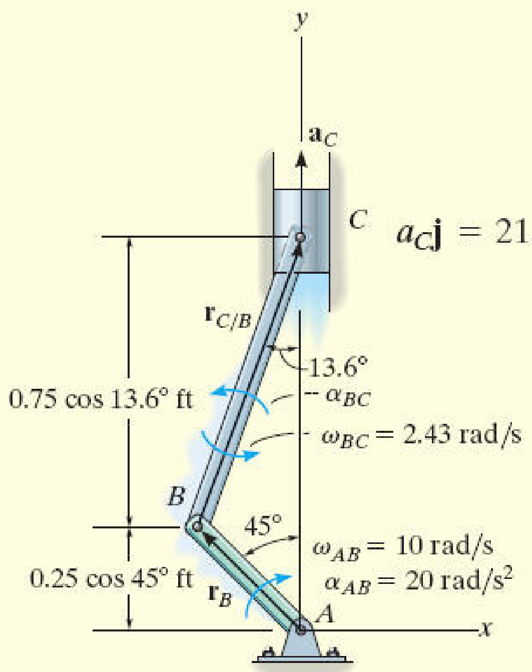
$$0 = 20.17 - 0.729\alpha_{BC}$$

$$a_C = 0.177\alpha_{BC} - 18.45$$

Solving yields

$$\alpha_{BC} = 27.7\ \text{rad/s}^2$$

$$a_C = -13.5\ \text{ft/s}^2 \qquad \textit{Ans.}$$



(b)

Fig. 16–31

NOTE: Since the piston is moving upward, the negative sign for a_C indicates that the piston is decelerating, i.e., $\mathbf{a}_C = \{-13.5\mathbf{j}\}$ ft/s^2. This causes the speed of the piston to decrease until *AB* becomes vertical, at which time the piston is momentarily at rest.

EXAMPLE 16.19

At the instant $\theta = 60°$, the rod in Fig. 16–33 has an angular velocity of 3 rad/s and an angular acceleration of 2 rad/s^2. At this same instant, collar C travels outward along the rod such that when $x = 0.2$ m the velocity is 2 m/s and the acceleration is 3 m/s^2, both measured relative to the rod. Determine the Coriolis acceleration and the velocity and acceleration of the collar at this instant.

SOLUTION

Coordinate Axes. The origin of both coordinate systems is located at point O, Fig. 16–33. Since motion of the collar is reported relative to the rod, the moving x, y, z frame of reference is *attached* to the rod.

Kinematic Equations.

$$\mathbf{v}_C = \mathbf{v}_O + \mathbf{\Omega} \times \mathbf{r}_{C/O} + (\mathbf{v}_{C/O})_{xyz} \tag{1}$$

$$\mathbf{a}_C = \mathbf{a}_O + \dot{\mathbf{\Omega}} \times \mathbf{r}_{C/O} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{C/O}) + 2\mathbf{\Omega} \times (\mathbf{v}_{C/O})_{xyz} + (\mathbf{a}_{C/O})_{xyz} \tag{2}$$

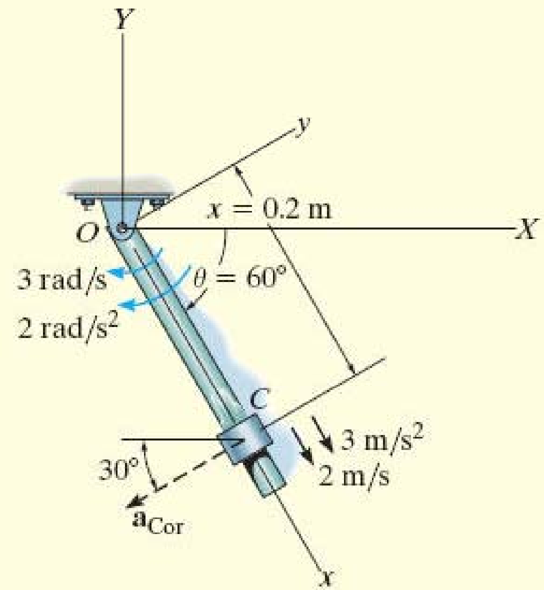


Fig. 16–33

EXAMPLE 16.19 CONTINUED

It will be simpler to express the data in terms of **i**, **j**, **k** component vectors rather than **I**, **J**, **K** components. Hence,

Motion of moving reference	Motion of C with respect to moving reference
$\mathbf{v}_O = \mathbf{0}$	$\mathbf{r}_{C/O} = \{0.2\mathbf{i}\}$ m
$\mathbf{a}_O = \mathbf{0}$	$(\mathbf{v}_{C/O})_{xyz} = \{2\mathbf{i}\}$ m/s
$\mathbf{\Omega} = \{-3\mathbf{k}\}$ rad/s	$(\mathbf{a}_{C/O})_{xyz} = \{3\mathbf{i}\}$ m/s^2
$\dot{\mathbf{\Omega}} = \{-2\mathbf{k}\}$ rad/s^2	

The Coriolis acceleration is defined as

$$\mathbf{a}_{\text{Cor}} = 2\mathbf{\Omega} \times (\mathbf{v}_{C/O})_{xyz} = 2(-3\mathbf{k}) \times (2\mathbf{i}) = \{-12\mathbf{j}\}\ \text{m/s}^2 \qquad \textit{Ans.}$$

This vector is shown dashed in Fig. 16–33. If desired, it may be resolved into **I**, **J** components acting along the X and Y axes, respectively.

The velocity and acceleration of the collar are determined by substituting the data into Eqs. 1 and 2 and evaluating the cross products, which yields

$$\begin{aligned}\mathbf{v}_C &= \mathbf{v}_O + \mathbf{\Omega} \times \mathbf{r}_{C/O} + (\mathbf{v}_{C/O})_{xyz}\\ &= \mathbf{0} + (-3\mathbf{k}) \times (0.2\mathbf{i}) + 2\mathbf{i}\\ &= \{2\mathbf{i} - 0.6\mathbf{j}\}\ \text{m/s} \qquad \textit{Ans.}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_C &= \mathbf{a}_O + \dot{\mathbf{\Omega}} \times \mathbf{r}_{C/O} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{C/O}) + 2\mathbf{\Omega} \times (\mathbf{v}_{C/O})_{xyz} + (\mathbf{a}_{C/O})_{xyz}\\ &= \mathbf{0} + (-2\mathbf{k}) \times (0.2\mathbf{i}) + (-3\mathbf{k}) \times [(-3\mathbf{k}) \times (0.2\mathbf{i})] + 2(-3\mathbf{k}) \times (2\mathbf{i}) + 3\mathbf{i}\\ &= \mathbf{0} - 0.4\mathbf{j} - 1.80\mathbf{i} - 12\mathbf{j} + 3\mathbf{i}\\ &= \{1.20\mathbf{i} - 12.4\mathbf{j}\}\ \text{m/s}^2 \qquad \textit{Ans.}\end{aligned}$$

EXAMPLE 16.20

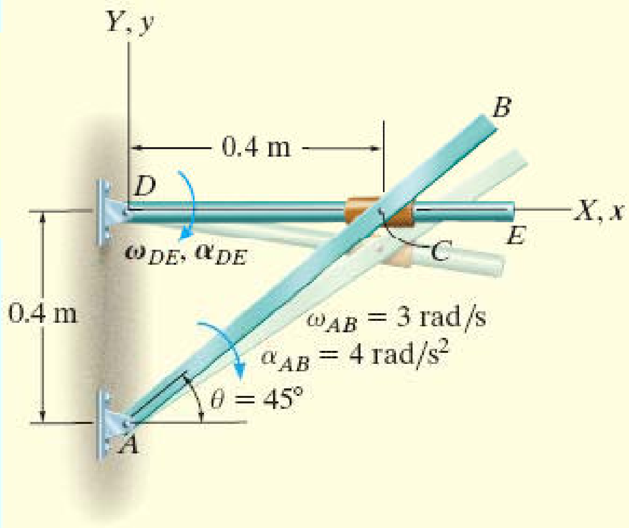


Fig. 16–34

Rod AB, shown in Fig. 16–34, rotates clockwise such that it has an angular velocity $\omega_{AB} = 3$ rad/s and angular acceleration $\alpha_{AB} = 4$ rad/s^2 when $\theta = 45°$. Determine the angular motion of rod DE at this instant. The collar at C is pin connected to AB and slides over rod DE.

SOLUTION

Coordinate Axes. The origin of both the fixed and moving frames of reference is located at D, Fig. 16–34. Furthermore, the x, y, z reference is attached to and rotates with rod DE so that the relative motion of the collar is easy to follow.

Kinematic Equations.

$$\mathbf{v}_C = \mathbf{v}_D + \Omega \times \mathbf{r}_{C/D} + (\mathbf{v}_{C/D})_{xyz} \quad (1)$$

$$\mathbf{a}_C = \mathbf{a}_D + \dot{\Omega} \times \mathbf{r}_{C/D} + \Omega \times (\Omega \times \mathbf{r}_{C/D}) + 2\Omega \times (\mathbf{v}_{C/D})_{xyz} + (\mathbf{a}_{C/D})_{xyz} \quad (2)$$

EXAMPLE 16.20 CONTINUED

All vectors will be expressed in terms of **i**, **j**, **k** components.

Motion of moving reference	Motion of C with respect to moving reference
$\mathbf{v}_D = \mathbf{0}$	$\mathbf{r}_{C/D} = \{0.4\mathbf{i}\}\text{m}$
$\mathbf{a}_D = \mathbf{0}$	$(\mathbf{v}_{C/D})_{xyz} = (v_{C/D})_{xyz}\mathbf{i}$
$\mathbf{\Omega} = -\omega_{DE}\mathbf{k}$	$(\mathbf{a}_{C/D})_{xyz} = (a_{C/D})_{xyz}\mathbf{i}$
$\dot{\mathbf{\Omega}} = -\alpha_{DE}\mathbf{k}$	

Motion of C: Since the collar moves along a *circular path* of radius AC, its velocity and acceleration can be determined using Eqs. 16–9 and 16–14.

$$\mathbf{v}_C = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{C/A} = (-3\mathbf{k}) \times (0.4\mathbf{i} + 0.4\mathbf{j}) = \{1.2\mathbf{i} - 1.2\mathbf{j}\}\text{ m/s}$$

$$\mathbf{a}_C = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{C/A} - \omega_{AB}^2\mathbf{r}_{C/A}$$
$$= (-4\mathbf{k}) \times (0.4\mathbf{i} + 0.4\mathbf{j}) - (3)^2(0.4\mathbf{i} + 0.4\mathbf{j}) = \{-2\mathbf{i} - 5.2\mathbf{j}\}\text{ m/s}^2$$

Substituting the data into Eqs. 1 and 2, we have

$$\mathbf{v}_C = \mathbf{v}_D + \mathbf{\Omega} \times \mathbf{r}_{C/D} + (\mathbf{v}_{C/D})_{xyz}$$
$$1.2\mathbf{i} - 1.2\mathbf{j} = \mathbf{0} + (-\omega_{DE}\mathbf{k}) \times (0.4\mathbf{i}) + (v_{C/D})_{xyz}\mathbf{i}$$
$$1.2\mathbf{i} - 1.2\mathbf{j} = \mathbf{0} - 0.4\omega_{DE}\mathbf{j} + (v_{C/D})_{xyz}\mathbf{i}$$
$$(v_{C/D})_{xyz} = 1.2\text{ m/s}$$
$$\omega_{DE} = 3\text{ rad/s} \circlearrowright \qquad \textit{Ans.}$$

$$\mathbf{a}_C = \mathbf{a}_D + \dot{\mathbf{\Omega}} \times \mathbf{r}_{C/D} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{C/D}) + 2\mathbf{\Omega} \times (\mathbf{v}_{C/D})_{xyz} + (\mathbf{a}_{C/D})_{xyz}$$
$$-2\mathbf{i} - 5.2\mathbf{j} = \mathbf{0} + (-\alpha_{DE}\mathbf{k}) \times (0.4\mathbf{i}) + (-3\mathbf{k}) \times [(-3\mathbf{k}) \times (0.4\mathbf{i})]$$
$$+ 2(-3\mathbf{k}) \times (1.2\mathbf{i}) + (a_{C/D})_{xyz}\mathbf{i}$$
$$-2\mathbf{i} - 5.2\mathbf{j} = -0.4\alpha_{DE}\mathbf{j} - 3.6\mathbf{i} - 7.2\mathbf{j} + (a_{C/D})_{xyz}\mathbf{i}$$
$$(a_{C/D})_{xyz} = 1.6\text{ m/s}^2$$
$$\alpha_{DE} = -5\text{ rad/s}^2 = 5\text{ rad/s}^2 \circlearrowleft \qquad \textit{Ans.}$$

EXAMPLE 16.21

Planes A and B fly at the same elevation and have the motions shown in Fig. 16–35. Determine the velocity and acceleration of A as measured by the pilot of B.

SOLUTION

Coordinate Axes. Since the relative motion of A with respect to the pilot in B is being sought, the x, y, z axes are attached to plane B, Fig. 16–35. At the *instant* considered, the origin B coincides with the origin of the fixed X, Y, Z frame.

Kinematic Equations.

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\Omega} \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz} \quad (1)$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{A/B} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{A/B}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz} \quad (2)$$

Motion of Moving Reference:

$$\mathbf{v}_B = \{600\mathbf{j}\} \text{ km/h}$$

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(600)^2}{400} = 900 \text{ km/h}^2$$

$$\mathbf{a}_B = (\mathbf{a}_B)_n + (\mathbf{a}_B)_t = \{900\mathbf{i} - 100\mathbf{j}\} \text{ km/h}^2$$

$$\Omega = \frac{v_B}{\rho} = \frac{600 \text{ km/h}}{400 \text{ km}} = 1.5 \text{ rad/h} \qquad \boldsymbol{\Omega} = \{-1.5\mathbf{k}\} \text{ rad/h}$$

$$\dot{\Omega} = \frac{(a_B)_t}{\rho} = \frac{100 \text{ km/h}^2}{400 \text{ km}} = 0.25 \text{ rad/h}^2 \qquad \dot{\boldsymbol{\Omega}} = \{0.25\mathbf{k}\} \text{ rad/h}^2$$

EXAMPLE 16.21 CONTINUED

Motion of A with Respect to Moving Reference:

$$\mathbf{r}_{A/B} = \{-4\mathbf{i}\} \text{ km} \quad (\mathbf{v}_{A/B})_{xyz} = ? \quad (\mathbf{a}_{A/B})_{xyz} = ?$$

Substituting the data into Eqs. 1 and 2, realizing that $\mathbf{v}_A = \{700\mathbf{j}\}\text{km/h}$ and $\mathbf{a}_A = \{50\mathbf{j}\} \text{ km/h}^2$, we have

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\Omega} \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz}$$

$$700\mathbf{j} = 600\mathbf{j} + (-1.5\mathbf{k}) \times (-4\mathbf{i}) + (\mathbf{v}_{A/B})_{xyz}$$

$$(\mathbf{v}_{A/B})_{xyz} = \{94\mathbf{j}\} \text{ km/h} \qquad \textit{Ans.}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{A/B} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{A/B}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$$

$$50\mathbf{j} = (900\mathbf{i} - 100\mathbf{j}) + (0.25\mathbf{k}) \times (-4\mathbf{i})$$
$$+ (-1.5\mathbf{k}) \times [(-1.5\mathbf{k}) \times (-4\mathbf{i})] + 2(-1.5\mathbf{k}) \times (94\mathbf{j}) + (\mathbf{a}_{A/B})_{xyz}$$

$$(\mathbf{a}_{A/B})_{xyz} = \{-1191\mathbf{i} + 151\mathbf{j}\} \text{ km/h}^2 \qquad \textit{Ans.}$$

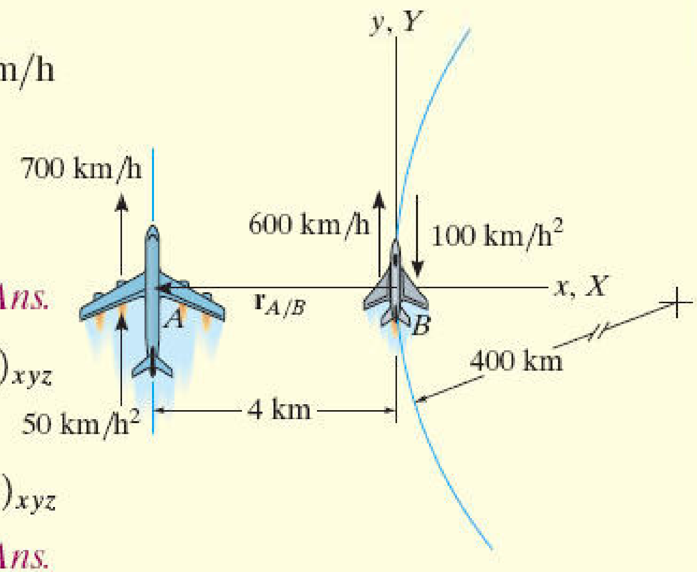


Fig. 16–35

NOTE: The solution of this problem should be compared with that of Example 12.26, where it is seen that $(v_{B/A})_{xyz} \neq (v_{A/B})_{xyz}$ and $(a_{B/A})_{xyz} \neq (a_{A/B})_{xyz}$.