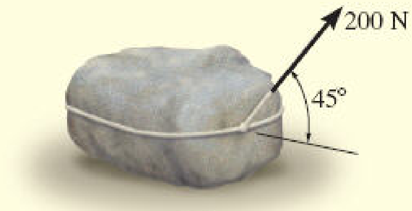
EXAMPLE 15.1

The 100-kg stone shown in Fig. 15–4*a* is originally at rest on the smooth horizontal surface. If a towing force of 200 N, acting at an angle of 45°, is applied to the stone for 10 s, determine the final velocity and the normal force which the surface exerts on the stone during this time interval.

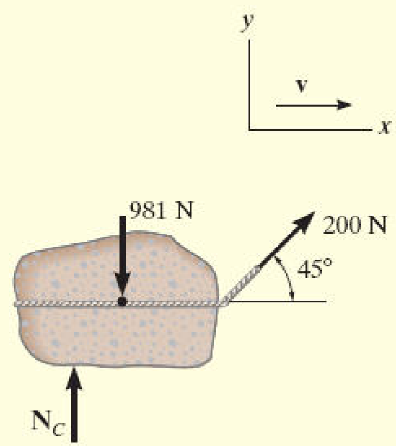


(a)

SOLUTION

This problem can be solved using the principle of impulse and momentum since it involves force, velocity, and time.

Free-Body Diagram. See Fig. 15–4*b*. Since all the forces acting are *constant*, the impulses are simply the product of the force magnitude and 10 s $[\mathbf{I} = \mathbf{F}_c(t_2 - t_1)]$. Note the alternative procedure of drawing the stone's impulse and momentum diagrams, Fig. 15–4*c*.



(b)

Principle of Impulse and Momentum. Applying Eqs. 15–4 yields

$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$0 + 200\text{ N}\cos 45^\circ(10\text{ s}) = (100\text{ kg})v_2$$

$$v_2 = 14.1\text{ m/s} \qquad \textit{Ans.}$$

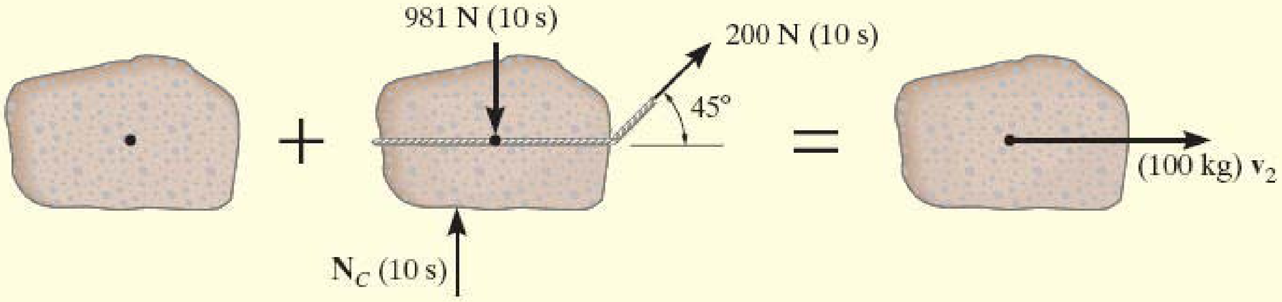
$$(+\uparrow) \qquad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$0 + N_C(10\text{ s}) - 981\text{ N}(10\text{ s}) + 200\text{ N}\sin 45^\circ(10\text{ s}) = 0$$

$$N_C = 840\text{ N} \qquad \textit{Ans.}$$

EXAMPLE 15.1 CONTINUED

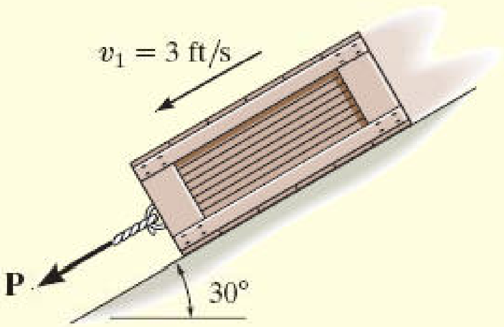
NOTE: Since no motion occurs in the y direction, direct application of the equilibrium equation $\Sigma F_y = 0$ gives the same result for N_C.



(c)

Fig. 15–4

EXAMPLE 15.2



(a)

The 50-lb crate shown in Fig. 15–5*a* is acted upon by a force having a variable magnitude $P = (20t)$ lb, where t is in seconds. Determine the crate's velocity 2 s after **P** has been applied. The initial velocity is $v_1 = 3$ ft/s down the plane, and the coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$.

SOLUTION

Free-Body Diagram. See Fig. 15–5*b*. Since the magnitude of force $P = 20t$ varies with time, the impulse it creates must be determined by integrating over the 2-s time interval.

Principle of Impulse and Momentum. Applying Eqs. 15–4 in the x direction, we have

$$(+\swarrow) \qquad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$\frac{50\text{ lb}}{32.2\text{ ft/s}^2}(3\text{ ft/s}) + \int_0^{2\text{ s}} 20t\,dt - 0.3N_C(2\text{ s}) + (50\text{ lb})\sin 30^\circ(2\text{ s}) = \frac{50\text{ lb}}{32.2\text{ ft/s}^2}v_2$$

$$4.658 + 40 - 0.6N_C + 50 = 1.553v_2$$

EXAMPLE 15.2 CONTINUED

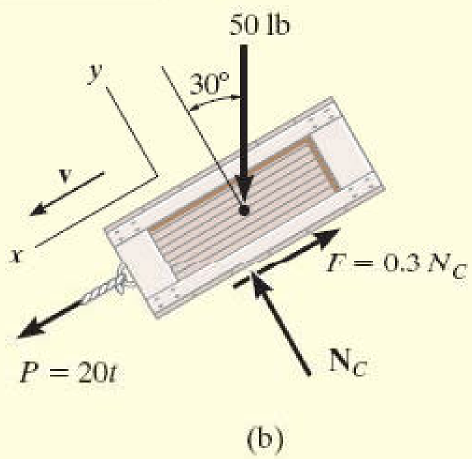


Fig. 15–5

The equation of equilibrium can be applied in the y direction. Why?

$$+\nwarrow \Sigma F_y = 0; \qquad N_C - 50\cos 30^\circ \text{ lb} = 0$$

Solving,

$$N_C = 43.30 \text{ lb}$$

$$v_2 = 44.2 \text{ ft/s} \swarrow \qquad \textit{Ans.}$$

NOTE: We can also solve this problem using the equation of motion. From Fig. 15–5*b*,

$$+\swarrow \Sigma F_x = ma_x; \; 20t - 0.3(43.30) + 50\sin 30^\circ = \frac{50}{32.2}a$$

$$a = 12.88t + 7.734$$

Using kinematics

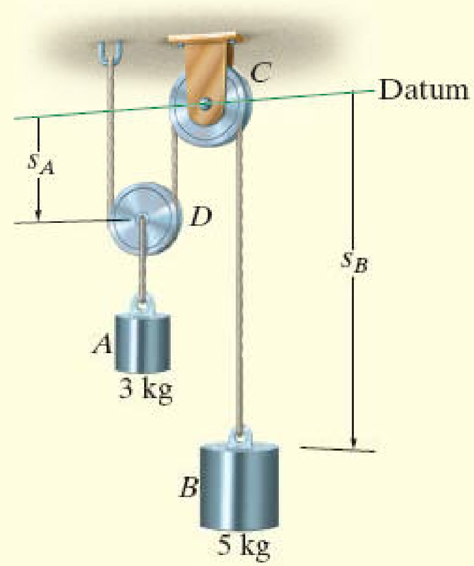
$$+\swarrow dv = a\,dt; \qquad \int_{3\text{ ft/s}}^{v} dv = \int_0^{2\text{ s}} (12.88t + 7.734)dt$$

$$v = 44.2 \text{ ft/s} \qquad \textit{Ans.}$$

By comparison, application of the principle of impulse and momentum eliminates the need for using kinematics ($a = dv/dt$) and thereby yields an easier method for solution.

EXAMPLE 15.3

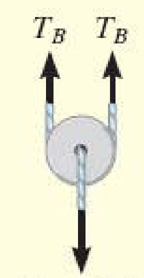
Blocks A and B shown in Fig. 15–6a have a mass of 3 kg and 5 kg, respectively. If the system is released from rest, determine the velocity of block B in 6 s. Neglect the mass of the pulleys and cord.



(a)

SOLUTION

Free-Body Diagram. See Fig. 15–6b. Since the weight of each block is constant, the cord tensions will also be constant. Furthermore, since the mass of pulley D is neglected, the cord tension $T_A = 2T_B$. Note that the blocks are both assumed to be moving downward in the positive coordinate directions, s_A and s_B.



Principle of Impulse and Momentum.

Block A:

$$(+\downarrow) \qquad m(v_A)_1 + \Sigma \int_{t_1}^{t_2} F_y\,dt = m(v_A)_2$$

$$0 - 2T_B(6\text{ s}) + 3(9.81)\text{ N}(6\text{ s}) = (3\text{ kg})(v_A)_2 \qquad (1)$$

Block B:

$$(+\downarrow) \qquad m(v_B)_1 + \Sigma \int_{t_1}^{t_2} F_y\,dt = m(v_B)_2$$

$$0 + 5(9.81)\text{ N}(6\text{ s}) - T_B(6\text{ s}) = (5\text{ kg})(v_B)_2 \qquad (2)$$

EXAMPLE 15.3 CONTINUED

Kinematics. Since the blocks are subjected to dependent motion, the velocity of A can be related to that of B by using the kinematic analysis discussed in Sec. 12.9. A horizontal datum is established through the fixed point at C, Fig. 15–6a, and the position coordinates, s_A and s_B, are related to the constant total length l of the vertical segments of the cord by the equation

$$2s_A + s_B = l$$

Taking the time derivative yields

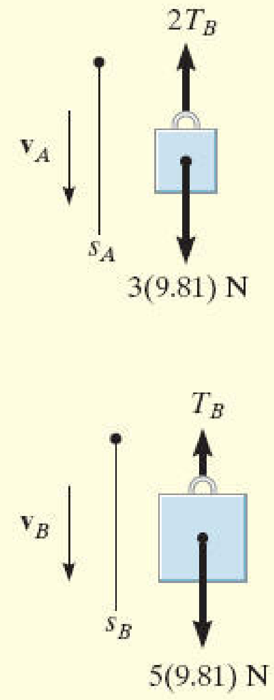
$$2v_A = -v_B \tag{3}$$

As indicated by the negative sign, when B moves downward A moves upward. Substituting this result into Eq. 1 and solving Eqs. 1 and 2 yields

$$(v_B)_2 = 35.8 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

$$T_B = 19.2 \text{ N}$$

NOTE: Realize that the *positive* (downward) direction for $\mathbf{v}_A$ and $\mathbf{v}_B$ is *consistent* in Figs. 15–6a and 15–6b and in Eqs. 1 to 3. This is important since we are seeking a simultaneous solution of equations.

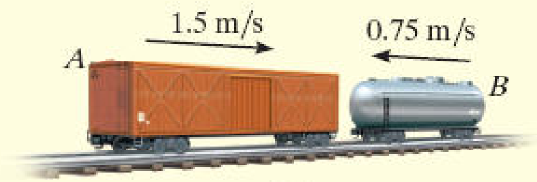


(b)

Fig. 15–6

EXAMPLE 15.4

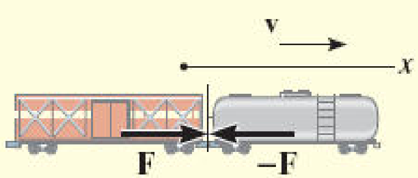
The 15-Mg boxcar A is coasting at 1.5 m/s on the horizontal track when it encounters a 12-Mg tank car B coasting at 0.75 m/s toward it as shown in Fig. 15–8*a*. If the cars collide and couple together, determine (a) the speed of both cars just after the coupling, and (b) the average force between them if the coupling takes place in 0.8 s.



(a)

SOLUTION

Part (a) Free-Body Diagram.* Here we have considered *both* cars as a single system, Fig. 15–8*b*. By inspection, momentum is conserved in the x direction since the coupling force $\mathbf{F}$ is *internal* to the system and will therefore cancel out. It is assumed both cars, when coupled, move at $\mathbf{v}_2$ in the positive x direction.



(b)

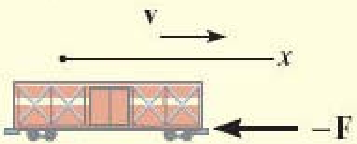
Conservation of Linear Momentum.

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$$(15\,000\text{ kg})(1.5\text{ m/s}) - 12\,000\text{ kg}(0.75\text{ m/s}) = (27\,000\text{ kg})v_2$$

$$v_2 = 0.5\text{ m/s} \rightarrow \qquad \textit{Ans.}$$

EXAMPLE 15.4 CONTINUED



(c)

Fig. 15–8

Part (b). The average (impulsive) coupling force, $\mathbf{F}_{avg}$, can be determined by applying the principle of linear momentum to *either one* of the cars.

Free-Body Diagram. As shown in Fig. 15–8*c*, by isolating the boxcar the coupling force is *external* to the car.

Principle of Impulse and Momentum. Since $\int F\,dt = F_{avg}\,\Delta t = F_{avg}(0.8\text{ s})$, we have

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + \Sigma\int F\,dt = m_A v_2$$

$$(15\,000\text{ kg})(1.5\text{ m/s}) - F_{avg}(0.8\text{ s}) = (15\,000\text{ kg})(0.5\text{ m/s})$$

$$F_{avg} = 18.8\text{ kN} \qquad \textit{Ans.}$$

NOTE: Solution was possible here since the boxcar's final velocity was obtained in Part (a). Try solving for F_{avg} by applying the principle of impulse and momentum to the tank car.

*Only horizontal forces are shown on the free-body diagram.

EXAMPLE 15.5

The 1200-lb cannon shown in Fig. 15–9*a* fires an 8-lb projectile with a muzzle velocity of 1500 ft/s relative to the ground. If firing takes place in 0.03 s, determine (a) the recoil velocity of the cannon just after firing, and (b) the average impulsive force acting on the projectile. The cannon support is fixed to the ground, and the horizontal recoil of the cannon is absorbed by two springs.

(a)

SOLUTION

Part (a) Free-Body Diagram.* As shown in Fig. 15–9*b*, we will consider the projectile and cannon as a single system, since the impulsive forces, $\mathbf{F}$, between the cannon and projectile are *internal* to the system and will therefore cancel from the analysis. Furthermore, during the time $\Delta t = 0.03$ s, the two recoil springs which are attached to the support each exert a *nonimpulsive force* $\mathbf{F}_s$ on the cannon. This is because Δt is very short, so that during this time the cannon only moves through a very small distance s. Consequently, $F_s = ks \approx 0$, where k is the spring's stiffness. Hence it can be concluded that momentum for the system is conserved in the *horizontal direction*.

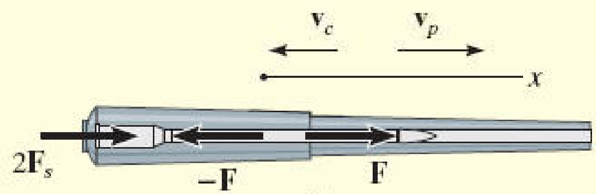
EXAMPLE 15.5 CONTINUED

Conservation of Linear Momentum.

$$(\overset{+}{\rightarrow}) \qquad m_c(v_c)_1 + m_p(v_p)_1 = -m_c(v_c)_2 + m_p(v_p)_2$$

$$0 + 0 = -\left(\frac{1200 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(v_c)_2 + \left(\frac{8 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(1500 \text{ ft/s})$$

$$(v_c)_2 = 10 \text{ ft/s} \leftarrow \qquad \textit{Ans.}$$



(b)

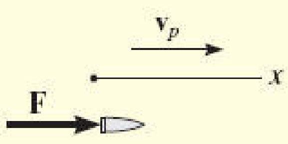
Part (b). The average impulsive force exerted by the cannon on the projectile can be determined by applying the principle of linear impulse and momentum to the projectile (or to the cannon). Why?

Principle of Impulse and Momentum. From Fig. 15–9*c*, with $\int F\,dt = F_{avg}\Delta t = F_{avg}(0.03)$, we have

$$(\overset{+}{\rightarrow}) \qquad m(v_p)_1 + \Sigma\int F\,dt = m(v_p)_2$$

$$0 + F_{avg}(0.03 \text{ s}) = \left(\frac{8 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(1500 \text{ ft/s})$$

$$F_{avg} = 12.4(10^3) \text{ lb} = 12.4 \text{ kip} \qquad \textit{Ans.}$$



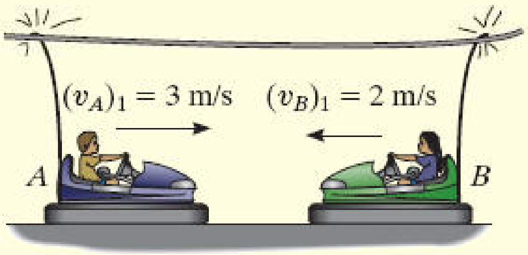
(c)

Fig. 15–9

NOTE: If the cannon is firmly fixed to its support (no springs), the reactive force of the support on the cannon must be considered as an external impulse to the system, since the support would allow no movement of the cannon.

*Only horizontal forces are shown on the free-body diagram.

EXAMPLE 15.6



(a)

The bumper cars A and B in Fig. 15–10a each have a mass of 150 kg and are coasting with the velocities shown before they freely collide head on. If no energy is lost during the collision, determine their velocities after collision.

SOLUTION

Free-Body Diagram. The cars will be considered as a single system. The free-body diagram is shown in Fig. 15–10b.

Conservation of Momentum.

$$(\xrightarrow{+}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$(150\text{ kg})(3\text{ m/s}) + (150\text{ kg})(-2\text{ m/s}) = (150\text{ kg})(v_A)_2 + (150\text{ kg})(v_B)_2$$

$$(v_A)_2 = 1 - (v_B)_2 \qquad (1)$$

EXAMPLE 15.6 CONTINUED

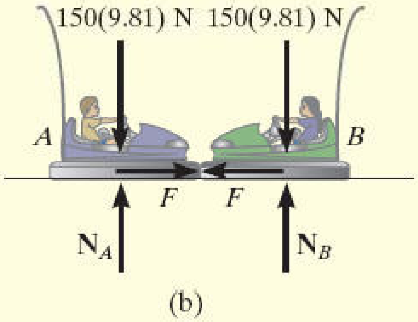


Fig. 15–10

Conservation of Energy. Since no energy is lost, the conservation of energy theorem gives

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}m_A(v_A)_1^2 + \frac{1}{2}m_B(v_B)_1^2 + 0 = \frac{1}{2}m_A(v_A)_2^2 + \frac{1}{2}m_B(v_B)_2^2 + 0$$

$$\frac{1}{2}(150\text{ kg})(3\text{ m/s})^2 + \frac{1}{2}(150\text{ kg})(2\text{ m/s})^2 + 0 = \frac{1}{2}(150\text{ kg})(v_A)_2^2$$

$$+ \frac{1}{2}(150\text{ kg})(v_B)_2{}^2 + 0$$

$$(v_A)_2^2 + (v_B)_2^2 = 13 \qquad (2)$$

Substituting Eq. (1) into (2) and simplifying, we get

$$(v_B)_2^2 - (v_B)_2 - 6 = 0$$

Solving for the two roots,

$$(v_B)_2 = 3\text{ m/s} \qquad \text{and} \qquad (v_B)_2 = -2\text{ m/s}$$

Since $(v_B)_2 = -2$ m/s refers to the velocity of B just *before* collision, then the velocity of B just after the collision must be

$$(v_B)_2 = 3\text{ m/s} \rightarrow \qquad \textit{Ans.}$$

Substituting this result into Eq. (1), we obtain

$$(v_A)_2 = 1 - 3\text{ m/s} = -2\text{ m/s} = 2\text{ m/s} \leftarrow \qquad \textit{Ans.}$$

EXAMPLE 15.7

An 800-kg rigid pile shown in Fig. 15–11*a* is driven into the ground using a 300-kg hammer. The hammer falls from rest at a height $y_0 = 0.5$ m and strikes the top of the pile. Determine the impulse which the pile exerts on the hammer if the pile is surrounded entirely by loose sand so that after striking, the hammer does *not* rebound off the pile.

SOLUTION

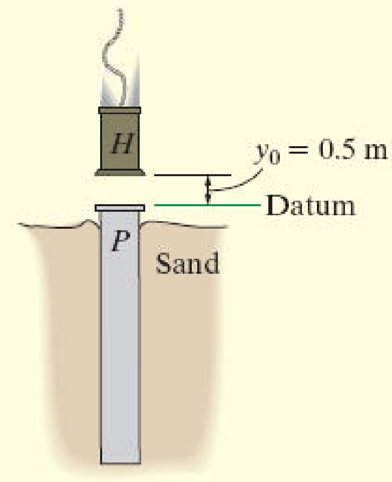
Conservation of Energy. The velocity at which the hammer strikes the pile can be determined using the conservation of energy equation applied to the hammer. With the datum at the top of the pile, Fig. 15–11*a*, we have

$$T_0 + V_0 = T_1 + V_1$$

$$\frac{1}{2}m_H(v_H)_0^2 + W_H y_0 = \frac{1}{2}m_H(v_H)_1^2 + W_H y_1$$

$$0 + 300(9.81)\text{ N}(0.5\text{ m}) = \frac{1}{2}(300\text{ kg})(v_H)_1^2 + 0$$

$$(v_H)_1 = 3.132\text{ m/s}$$



(a)

Free-Body Diagram. From the physical aspects of the problem, the free-body diagram of the hammer and pile, Fig. 15–11*b*, indicates that during the *short time* from *just before* to *just after* the *collision*, the weights of the hammer and pile and the resistance force $\mathbf{F}_s$ of the sand are all *nonimpulsive*. The impulsive force $\mathbf{R}$ is internal to the system and therefore cancels. Consequently, momentum is conserved in the vertical direction during this short time.

EXAMPLE 15.7 CONTINUED

Conservation of Momentum. Since the hammer does not rebound off the pile just after collision, then $(v_H)_2 = (v_P)_2 = v_2$.

$$(+\downarrow) \qquad m_H(v_H)_1 + m_P(v_P)_1 = m_H v_2 + m_P v_2$$

$$(300\text{ kg})(3.132\text{ m/s}) + 0 = (300\text{ kg})v_2 + (800\text{ kg})v_2$$

$$v_2 = 0.8542\text{ m/s}$$

Principle of Impulse and Momentum. The impulse which the pile imparts to the hammer can now be determined since $\mathbf{v}_2$ is known. From the free-body diagram for the hammer, Fig. 15–11*c*, we have

$$(+\downarrow) \qquad m_H(v_H)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m_H v_2$$

$$(300\text{ kg})(3.132\text{ m/s}) - \int R\,dt = (300\text{ kg})(0.8542\text{ m/s})$$

$$\int R\,dt = 683\text{ N}\cdot\text{s} \qquad \textit{Ans.}$$

NOTE: The equal but opposite impulse acts on the pile. Try finding this impulse by applying the principle of impulse and momentum to the pile.

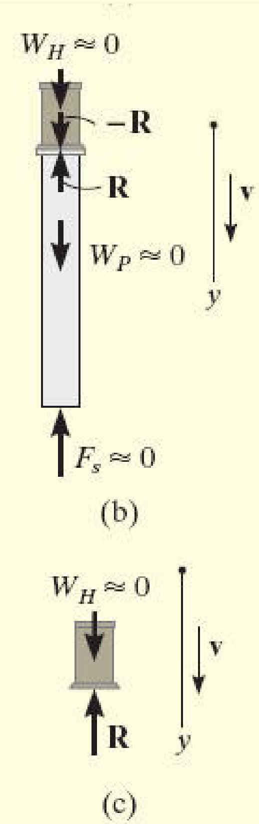
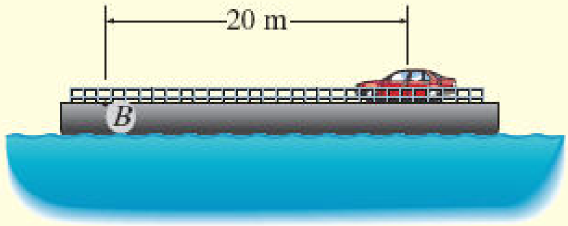
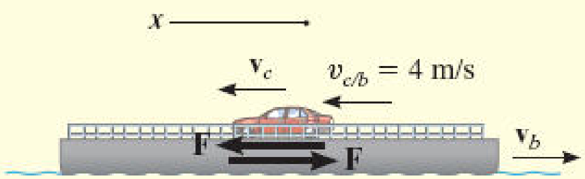


Fig. 15–11

EXAMPLE 15.8



(a)



(b)

Fig. 15–12

The 1.5-Mg car in Fig. 15–12*a* moves on the 10-Mg barge to the left with a constant speed of 4 m/s, measured relative to the barge. Neglecting water resistance, determine the velocity of the barge and the displacement of the barge when the car reaches point *B*. Initially, the car and the barge are at rest relative to the water.

SOLUTION

Free-Body Diagram. If the car and the barge are considered as a single system, the traction force between the car and the barge becomes internal to the system, and so linear momentum will be conserved along the x axis, Fig. 15–12*b*.

Conservation of Momentum. When writing the conservation of momentum equation, it is important that the velocities be measured from the same inertial coordinate system, assumed here to be fixed. We will also assume that as the car goes to the left the barge goes to the right, as shown in Fig. 15–12*b*.

Applying the conservation of linear momentum to the car and barge system,

$$(\overset{+}{\leftarrow}) \qquad 0 + 0 = m_c v_c - m_b v_b$$

$$0 = (1.5(10^3)\text{ kg})v_c - (10(10^3)\text{ kg})v_b$$

$$1.5v_c - 10v_b = 0 \qquad (1)$$

EXAMPLE 15.8 CONTINUED

Kinematics. Since the velocity of the car relative to the barge is known, then the velocity of the car and the barge can also be related using the relative velocity equation.

$$(\overset{+}{\leftarrow}) \qquad \mathbf{v}_c = \mathbf{v}_b + \mathbf{v}_{c/b}$$

$$v_c = -v_b + 4 \text{ m/s} \qquad (2)$$

Solving Eqs. (1) and (2),

$$v_b = 0.5217 \text{ m/s} = 0.522 \text{ m/s} \rightarrow \qquad \textit{Ans.}$$

$$v_c = 3.478 \text{ m/s} \leftarrow$$

The car travels $s_{c/b} = 20$ m on the barge at a constant relative speed of 4 m/s. Thus, the time for the car to reach point B is

$$s_{c/b} = v_{c/b}\, t$$

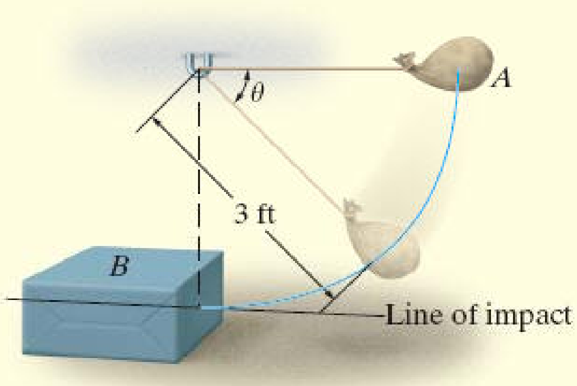
$$20 \text{ m} = (4 \text{ m/s})\, t$$

$$t = 5 \text{ s}$$

The displacement of the barge is therefore

$$(\overset{+}{\rightarrow}) \qquad s_b = v_b t = 0.5217 \text{ m/s}(5 \text{ s}) = 2.61 \text{ m} \rightarrow \qquad \textit{Ans.}$$

EXAMPLE 15.9



(a)

The bag A, having a weight of 6 lb, is released from rest at the position $\theta = 0°$, as shown in Fig. 15–16*a*. After falling to $\theta = 90°$, it strikes an 18-lb box B. If the coefficient of restitution between the bag and box is $e = 0.5$, determine the velocities of the bag and box just after impact. What is the loss of energy during collision?

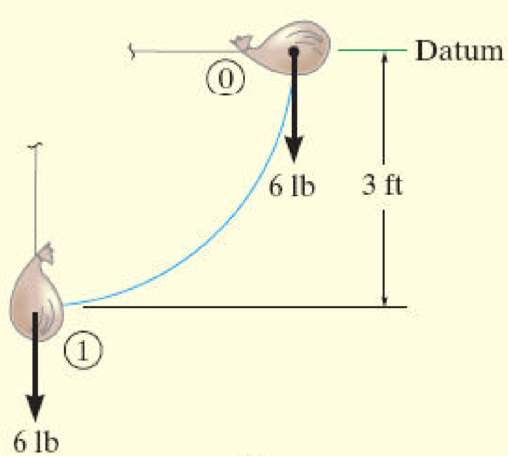
SOLUTION

This problem involves central impact. Why? Before analyzing the mechanics of the impact, however, it is first necessary to obtain the velocity of the bag *just before* it strikes the box.

Conservation of Energy. With the datum at $\theta = 0°$, Fig. 15–16*b*, we have

$$T_0 + V_0 = T_1 + V_1$$

$$0 + 0 = \frac{1}{2}\left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_A)_1^2 - 6\text{ lb}(3\text{ ft}); \qquad (v_A)_1 = 13.90\text{ ft/s}$$



(b)

Conservation of Momentum. After impact we will assume A and B travel to the left. Applying the conservation of momentum to the system, Fig. 15–16*c*, we have

$$(\overset{+}{\leftarrow}) \qquad m_B(v_B)_1 + m_A(v_A)_1 = m_B(v_B)_2 + m_A(v_A)_2$$

$$0 + \left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(13.90\text{ ft/s}) = \left(\frac{18\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_B)_2 + \left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(v_A)_2$$

$$(v_A)_2 = 13.90 - 3(v_B)_2 \qquad (1)$$

EXAMPLE 15.9 CONTINUED

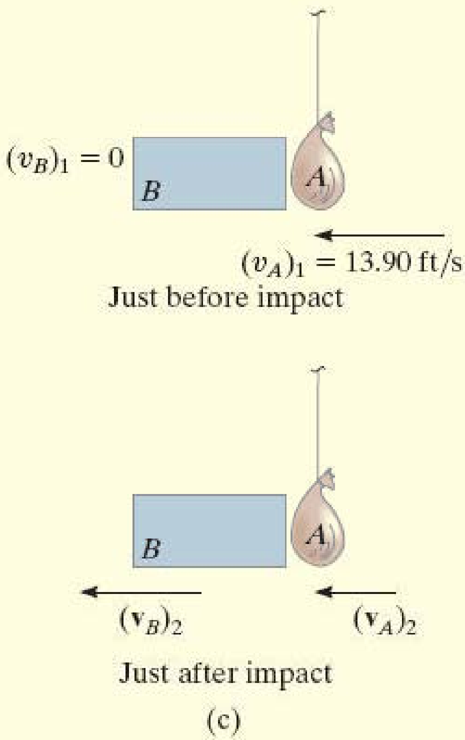


Fig. 15–16

Coefficient of Restitution. Realizing that for separation to occur after collision $(v_B)_2 > (v_A)_2$, Fig. 15–16*c*, we have

$$(\overset{+}{\leftarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \quad 0.5 = \frac{(v_B)_2 - (v_A)_2}{13.90\text{ ft/s} - 0}$$

$$(v_A)_2 = (v_B)_2 - 6.950 \qquad (2)$$

Solving Eqs. 1 and 2 simultaneously yields

$$(v_A)_2 = -1.74\text{ ft/s} = 1.74\text{ ft/s} \rightarrow \quad \text{and} \quad (v_B)_2 = 5.21\text{ ft/s} \leftarrow \qquad \textit{Ans.}$$

Loss of Energy. Applying the principle of work and energy to the bag and box just before and just after collision, we have

$\Sigma U_{1-2} = T_2 - T_1;$

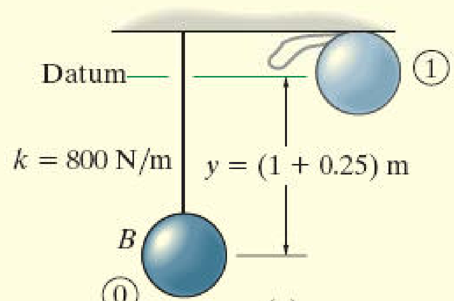
$$\Sigma U_{1-2} = \left[\frac{1}{2}\left(\frac{18\text{ lb}}{32.2\text{ ft/s}^2}\right)(5.21\text{ ft/s})^2 + \frac{1}{2}\left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(1.74\text{ ft/s})^2\right] - \left[\frac{1}{2}\left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right)(13.9\text{ ft/s})^2\right]$$

$$\Sigma U_{1-2} = -10.1\text{ ft}\cdot\text{lb} \qquad \textit{Ans.}$$

NOTE: The energy loss occurs due to inelastic deformation during the collision.

EXAMPLE 15.10

Ball B shown in Fig. 15–17a has a mass of 1.5 kg and is suspended from the ceiling by a 1-m-long elastic cord. If the cord is *stretched* downward 0.25 m and the ball is released from rest, determine how far the cord stretches after the ball rebounds from the ceiling. The stiffness of the cord is $k = 800$ N/m, and the coefficient of restitution between the ball and ceiling is $e = 0.8$. The ball makes a central impact with the ceiling.



(a)

SOLUTION

First we must obtain the velocity of the ball *just before* it strikes the ceiling using energy methods, then consider the impulse and momentum between the ball and ceiling, and finally again use energy methods to determine the stretch in the cord.

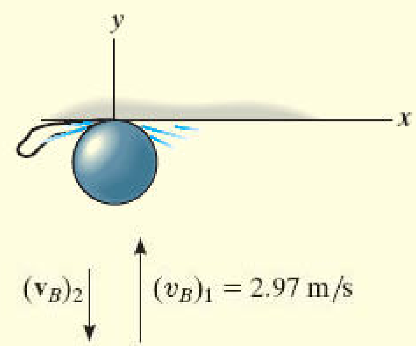
Conservation of Energy. With the datum located as shown in Fig. 15–17a, realizing that initially $y = y_0 = (1 + 0.25)$ m $= 1.25$ m, we have

$$T_0 + V_0 = T_1 + V_1$$

$$\tfrac{1}{2}m(v_B)_0^2 - W_B y_0 + \tfrac{1}{2}ks^2 = \tfrac{1}{2}m(v_B)_1^2 + 0$$

$$0 - 1.5(9.81)\text{N}(1.25\text{ m}) + \tfrac{1}{2}(800\text{ N/m})(0.25\text{ m})^2 = \tfrac{1}{2}(1.5\text{ kg})(v_B)_1^2$$

$$(v_B)_1 = 2.968\text{ m/s}\uparrow$$



(b)

The interaction of the ball with the ceiling will now be considered using the principles of impact.* Since an unknown portion of the mass of the ceiling is involved in the impact, the conservation of momentum for the ball–ceiling system will not be written. The "velocity" of this portion of ceiling is zero since it (or the earth) are assumed to remain at rest both before and after impact.

EXAMPLE 15.10 CONTINUED

Coefficient of Restitution. Fig. 15–17*b*.

$$(+\uparrow) \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \qquad 0.8 = \frac{(v_B)_2 - 0}{0 - 2.968 \text{ m/s}}$$

$$(v_B)_2 = -2.374 \text{ m/s} = 2.374 \text{ m/s} \downarrow$$

Conservation of Energy. The maximum stretch s_3 in the cord can be determined by again applying the conservation of energy equation to the ball just after collision. Assuming that $y = y_3 = (1 + s_3)$ m, Fig. 15–17*c*, then

$$T_2 + V_2 = T_3 + V_3$$

$$\tfrac{1}{2}m(v_B)_2^2 + 0 = \tfrac{1}{2}m(v_B)_3^2 - W_B y_3 + \tfrac{1}{2}ks_3^2$$

$$\tfrac{1}{2}(1.5 \text{ kg})(2.37 \text{ m/s})^2 = 0 - 9.81(1.5) \text{ N}(1 \text{ m} + s_3) + \tfrac{1}{2}(800 \text{ N/m})s_3^2$$

$$400s_3^2 - 14.715s_3 - 18.94 = 0$$

Solving this quadratic equation for the positive root yields

$$s_3 = 0.237 \text{ m} = 237 \text{ mm} \qquad \textit{Ans.}$$

* The weight of the ball is considered a nonimpulsive force.

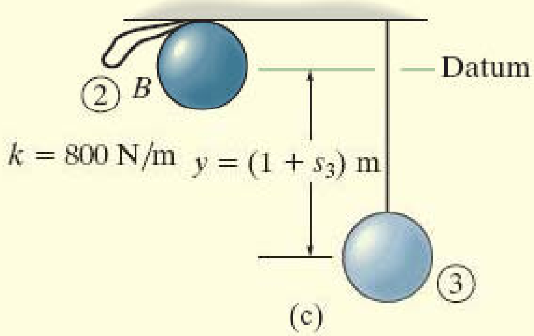
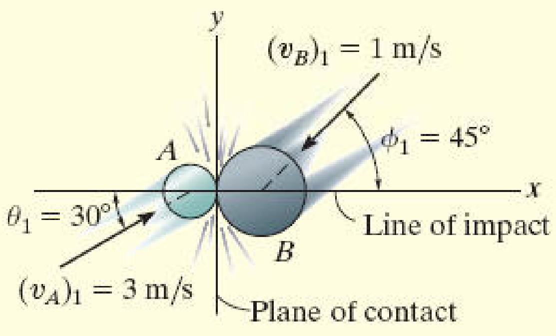


Fig. 15–17

EXAMPLE 15.11



(a)

Two smooth disks A and B, having a mass of 1 kg and 2 kg, respectively, collide with the velocities shown in Fig. 15–18*a*. If the coefficient of restitution for the disks is $e = 0.75$, determine the x and y components of the final velocity of each disk just after collision.

SOLUTION

This problem involves *oblique impact*. Why? In order to solve it, we have established the x and y axes along the line of impact and the plane of contact, respectively, Fig. 15–18*a*.

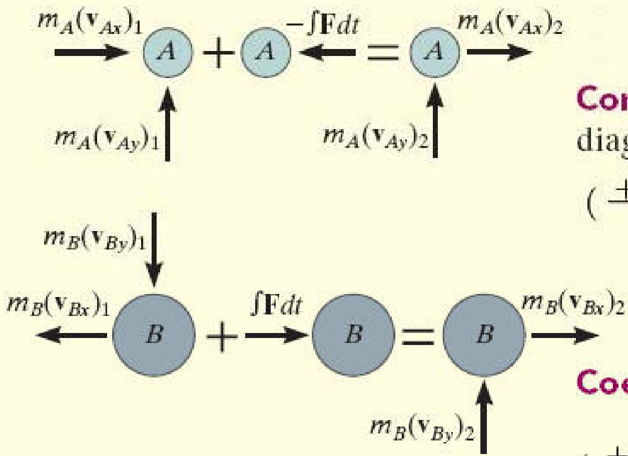
Resolving each of the initial velocities into x and y components, we have

$$(v_{Ax})_1 = 3\cos 30° = 2.598 \text{ m/s} \qquad (v_{Ay})_1 = 3\sin 30° = 1.50 \text{ m/s}$$

$$(v_{Bx})_1 = -1\cos 45° = -0.7071 \text{ m/s} \quad (v_{By})_1 = -1\sin 45° = -0.7071 \text{ m/s}$$

The four unknown velocity components after collision are *assumed to act in the positive directions*, Fig. 15–18*b*. Since the impact occurs in the x direction (line of impact), the conservation of momentum for *both* disks can be applied in this direction. Why?

EXAMPLE 15.11 CONTINUED



(b)

Conservation of "x" Momentum. In reference to the momentum diagrams, we have

$$(\xrightarrow{+})\qquad m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$$

$$1\text{ kg}(2.598\text{ m/s}) + 2\text{ kg}(-0.707\text{ m/s}) = 1\text{ kg}(v_{Ax})_2 + 2\text{ kg}(v_{Bx})_2$$

$$(v_{Ax})_2 + 2(v_{Bx})_2 = 1.184 \qquad (1)$$

Coefficient of Restitution (x).

$$(\xrightarrow{+})\qquad e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1};\quad 0.75 = \frac{(v_{Bx})_2 - (v_{Ax})_2}{2.598\text{ m/s} - (-0.7071\text{ m/s})}$$

$$(v_{Bx})_2 - (v_{Ax})_2 = 2.479 \qquad (2)$$

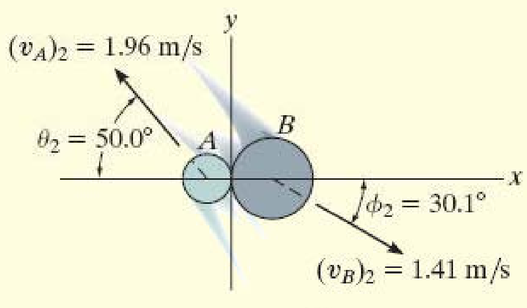
Solving Eqs. 1 and 2 for $(v_{Ax})_2$ and $(v_{Bx})_2$ yields

$(v_{Ax})_2 = -1.26\text{ m/s} = 1.26\text{ m/s} \leftarrow \qquad (v_{Bx})_2 = 1.22\text{ m/s} \rightarrow$ *Ans.*

Conservation of "y" Momentum. The momentum of *each disk* is *conserved* in the *y* direction (plane of contact), since the disks are smooth and therefore *no* external impulse acts in this direction. From Fig. 15–18*b*,

$(+\uparrow)\ m_A(v_{Ay})_1 = m_A(v_{Ay})_2;\ (v_{Ay})_2 = 1.50\text{ m/s} \uparrow$ *Ans.*

$(+\uparrow)\ m_B(v_{By})_1 = m_B(v_{By})_2;\ (v_{By})_2 = -0.707\text{ m/s} = 0.707\text{ m/s} \downarrow$ *Ans.*



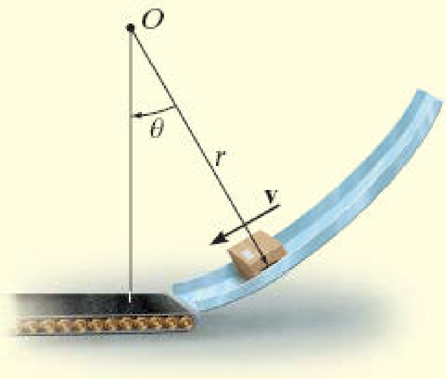
(c)

Fig. 15–18

NOTE: Show that when the velocity components are summed vectorially, one obtains the results shown in Fig. 15–18*c*.

EXAMPLE 15.12

The box shown in Fig. 15–22*a* has a mass m and travels down the smooth circular ramp such that when it is at the angle θ it has a speed v. Determine its angular momentum about point O at this instant and the rate of increase in its speed, i.e., a_t.



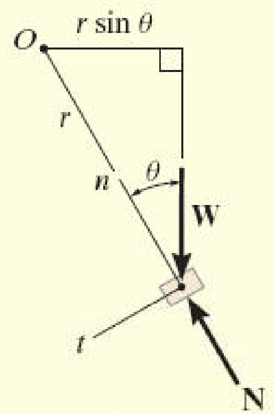


Fig. 15–22

EXAMPLE 15.12 CONTINUED

SOLUTION

Since $\mathbf{v}$ is tangent to the path, applying Eq. 15–12 the angular momentum is

$$H_O = rmv \circlearrowright \qquad \textit{Ans.}$$

The rate of increase in its speed (dv/dt) can be found by applying Eq. 15–15. From the free-body diagram of the box, Fig. 15–22*b*, it can be seen that only the weight $W = mg$ contributes a moment about point O. We have

$$\circlearrowleft + \Sigma M_O = \dot{H}_O; \qquad mg(r \sin\theta) = \frac{d}{dt}(rmv)$$

Since r and m are constant,

$$mgr \sin\theta = rm\frac{dv}{dt}$$

$$\frac{dv}{dt} = g \sin\theta \qquad \textit{Ans.}$$

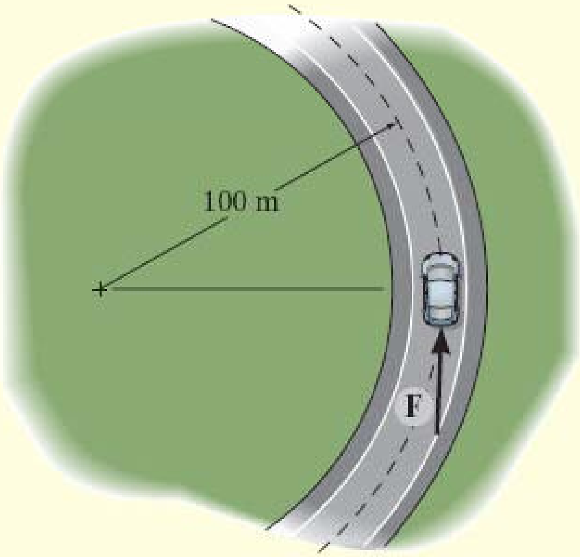
NOTE: This same result can, of course, be obtained from the equation of motion applied in the tangential direction, Fig. 15–22*b*, i.e.,

$$+\swarrow \Sigma F_t = ma_t; \qquad mg \sin\theta = m\left(\frac{dv}{dt}\right)$$

$$\frac{dv}{dt} = g \sin\theta \qquad \textit{Ans.}$$

EXAMPLE 15.13

The 1.5-Mg car travels along the circular road as shown in Fig. 15–24*a*. If the traction force of the wheels on the road is $F = (150t^2)$ N, where t is in seconds, determine the speed of the car when $t = 5$ s. The car initially travels with a speed of 5 m/s. Negect the size of the car.



(a)

EXAMPLE 15.13 CONTINUED

SOLUTION

Free-Body Diagram. The free-body diagram of the car is shown in Fig. 15–24*b*. If we apply the principle of angular impulse and momentum about the z axis, then the angular impulse created by the weight, normal force, and radial frictional force will be eliminated since they act parallel to the axis or pass through it.

Principle of Angular Impulse and Momentum.

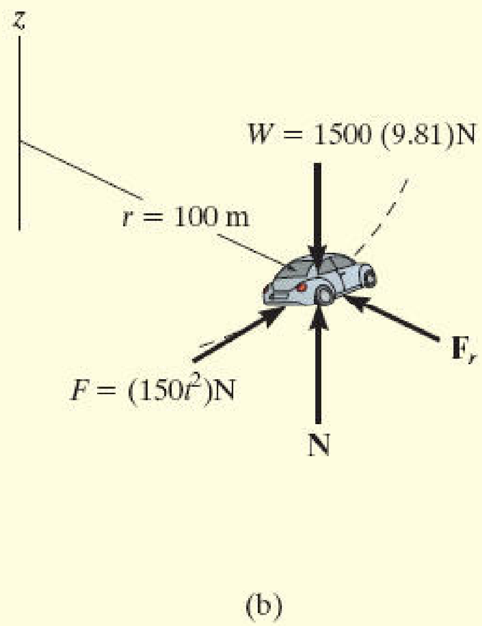
$$(H_z)_1 + \Sigma \int_{t_1}^{t_2} M_z\,dt = (H_z)_2$$

$$rm_c(v_c)_1 + \int_{t_1}^{t_2} rF\,dt = rm_c(v_c)_2$$

$$(100\text{ m})(1500\text{ kg})(5\text{ m/s}) + \int_0^{5\text{ s}} (100\text{ m})[(150t^2)\text{ N}]\,dt$$
$$= (100\text{ m})(1500\text{ kg})(v_c)_2$$

$$750(10^3) + 5000t^3\Big|_0^{5\text{ s}} = 150(10^3)(v_c)_2$$

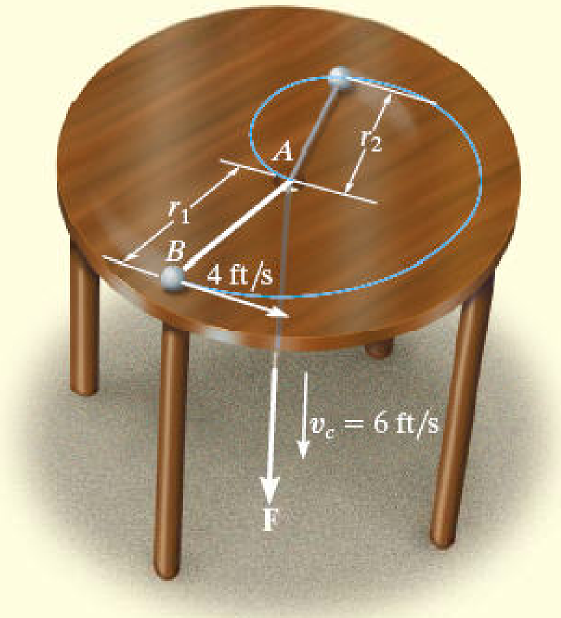
$$(v_c)_2 = 9.17\text{ m/s} \qquad \textit{Ans.}$$



(b)

Fig. 15–24

EXAMPLE 15.14



(a)

The 0.8-lb ball B, shown in Fig. 15–25a, is attached to a cord which passes through a hole at A in a smooth table. When the ball is $r_1 = 1.75$ ft from the hole, it is rotating around in a circle such that its speed is $v_1 = 4$ ft/s. By applying the force $\mathbf{F}$ the cord is pulled downward through the hole with a constant speed $v_c = 6$ ft/s. Determine (a) the speed of the ball at the instant it is $r_2 = 0.6$ ft from the hole, and (b) the amount of work done by $\mathbf{F}$ in shortening the radial distance from r_1 to r_2. Neglect the size of the ball.

SOLUTION

Part (a) Free-Body Diagram. As the ball moves from r_1 to r_2, Fig. 15–25b, the cord force $\mathbf{F}$ on the ball always passes through the z axis, and the weight and $\mathbf{N}_B$ are parallel to it. Hence the moments, or angular impulses created by these forces, are all *zero* about this axis. Therefore, angular momentum is conserved about the z axis.

EXAMPLE 15.14 CONTINUED

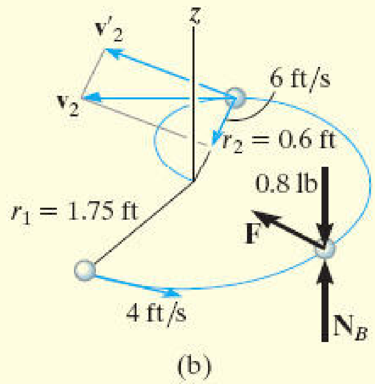


Fig. 15–25

Conservation of Angular Momentum. The ball's velocity $\mathbf{v}_2$ is resolved into two components. The radial component, 6 ft/s, is known; however, it produces zero angular momentum about the z axis. Thus,

$$\mathbf{H}_1 = \mathbf{H}_2$$

$$r_1 m_B v_1 = r_2 m_B v_2'$$

$$1.75 \text{ ft}\left(\frac{0.8 \text{ lb}}{32.2 \text{ ft/s}^2}\right) 4 \text{ ft/s} = 0.6 \text{ ft}\left(\frac{0.8 \text{ lb}}{32.2 \text{ ft/s}^2}\right) v_2'$$

$$v_2' = 11.67 \text{ ft/s}$$

The speed of the ball is thus

$$v_2 = \sqrt{(11.67 \text{ ft/s})^2 + (6 \text{ ft/s})^2}$$
$$= 13.1 \text{ ft/s}$$

Part (b). The only force that does work on the ball is **F**. (The normal force and weight do not move vertically.) The initial and final kinetic energies of the ball can be determined so that from the principle of work and energy we have

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{0.8 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(4 \text{ ft/s})^2 + U_F = \frac{1}{2}\left(\frac{0.8 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(13.1 \text{ ft/s})^2$$

$$U_F = 1.94 \text{ ft} \cdot \text{lb} \qquad \textit{Ans.}$$

NOTE: The force F is not constant because the normal component of acceleration, $a_n = v^2/r$, changes as r changes.

EXAMPLE 15.15

The 2-kg disk shown in Fig. 15–26*a* rests on a smooth horizontal surface and is attached to an elastic cord that has a stiffness $k_c = 20$ N/m and is initially unstretched. If the disk is given a velocity $(v_D)_1 = 1.5$ m/s, perpendicular to the cord, determine the rate at which the cord is being stretched and the speed of the disk at the instant the cord is stretched 0.2 m.

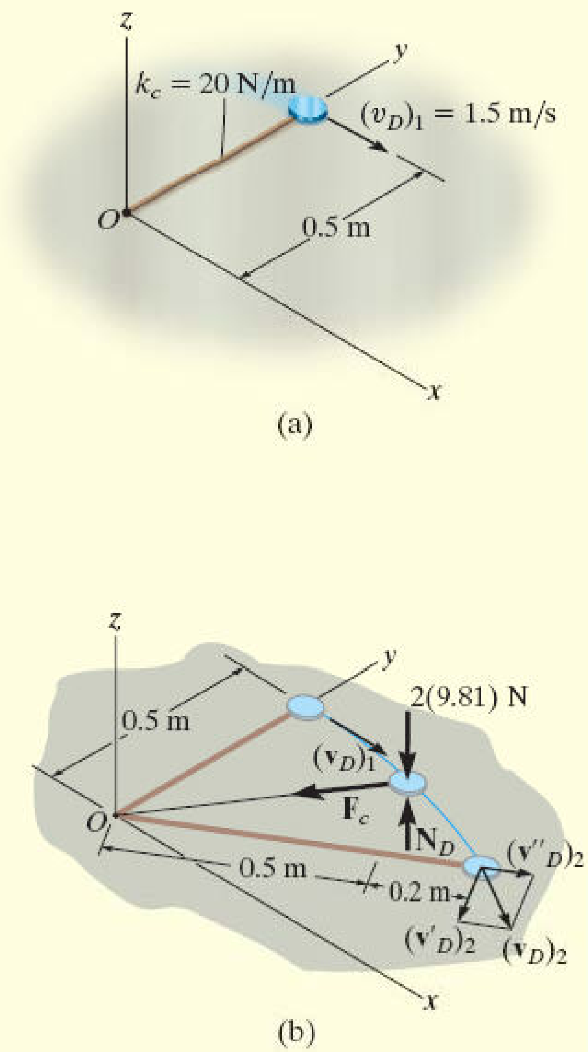


Fig. 15–26

SOLUTION

Free-Body Diagram. After the disk has been launched, it slides along the path shown in Fig. 15–26*b*. By inspection, angular momentum about point O (or the z axis) is *conserved*, since none of the forces produce an angular impulse about this axis. Also, when the distance is 0.7 m, only the transverse component $(\mathbf{v}'_D)_2$ produces angular momentum of the disk about O.

Conservation of Angular Momentum. The component $(\mathbf{v}'_D)_2$ can be obtained by applying the conservation of angular momentum about O (the z axis).

$$(\mathbf{H}_O)_1 = (\mathbf{H}_O)_2$$

$$r_1 m_D (v_D)_1 = r_2 m_D (v'_D)_2$$

$$0.5 \text{ m } (2 \text{ kg})(1.5 \text{ m/s}) = 0.7 \text{ m}(2 \text{ kg})(v'_D)_2$$

$$(v'_D)_2 = 1.071 \text{ m/s}$$

EXAMPLE 15.15 CONTINUED

Conservation of Energy. The speed of the disk can be obtained by applying the conservation of energy equation at the point where the disk was launched and at the point where the cord is stretched 0.2 m.

$$T_1 + V_1 = T_2 + V_2$$

$$\tfrac{1}{2}m_D(v_D)_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}m_D(v_D)_2^2 + \tfrac{1}{2}kx_2^2$$

$$\tfrac{1}{2}(2\text{ kg})(1.5\text{ m/s})^2 + 0 = \tfrac{1}{2}(2\text{ kg})(v_D)_2^2 + \tfrac{1}{2}(20\text{ N/m})(0.2\text{ m})^2$$

$$(v_D)_2 = 1.360\text{ m/s} = 1.36\text{ m/s} \qquad \textit{Ans.}$$

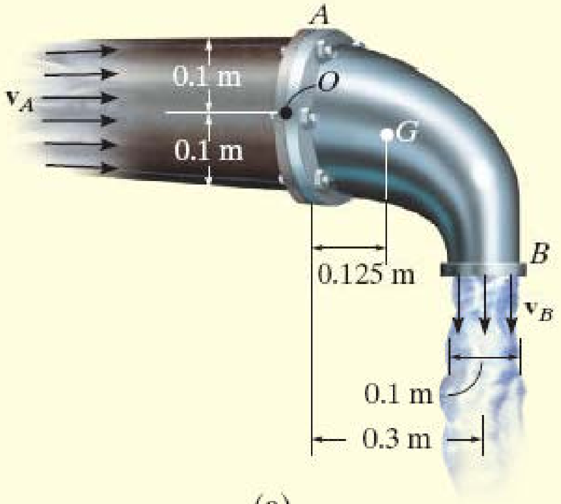
Having determined $(v_D)_2$ and its component $(v'_D)_2$, the rate of stretch of the cord, or radial component, $(v''_D)_2$ is determined from the Pythagorean theorem,

$$(v''_D)_2 = \sqrt{(v_D)_2^2 - (v'_D)_2^2}$$

$$= \sqrt{(1.360\text{ m/s})^2 - (1.071\text{ m/s})^2}$$

$$= 0.838\text{ m/s} \qquad \textit{Ans.}$$

EXAMPLE 15.16



(a)

Determine the components of reaction which the fixed pipe joint at A exerts on the elbow in Fig. 15–28a, if water flowing through the pipe is subjected to a static gauge pressure of 100 kPa at A. The discharge at B is $Q_B = 0.2\ \text{m}^3/\text{s}$. Water has a density $\rho_w = 1000\ \text{kg/m}^3$, and the water-filled elbow has a mass of 20 kg and center of mass at G.

SOLUTION

We will consider the control volume to be the outer surface of the elbow. Using a fixed inertial coordinate system, the velocity of flow at A and B and the mass flow rate can be obtained from Eq. 15–27. Since the density of water is constant, $Q_B = Q_A = Q$. Hence,

$$\frac{dm}{dt} = \rho_w Q = (1000\ \text{kg/m}^3)(0.2\ \text{m}^3/\text{s}) = 200\ \text{kg/s}$$

$$v_B = \frac{Q}{A_B} = \frac{0.2\ \text{m}^3/\text{s}}{\pi(0.05\ \text{m})^2} = 25.46\ \text{m/s} \downarrow$$

$$v_A = \frac{Q}{A_A} = \frac{0.2\ \text{m}^3/\text{s}}{\pi(0.1\ \text{m})^2} = 6.37\ \text{m/s} \rightarrow$$

EXAMPLE 15.16 CONTINUED

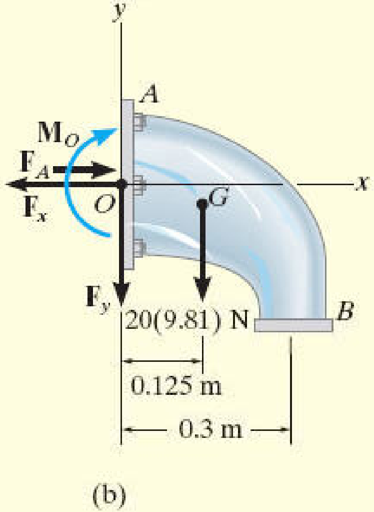


Fig. 15–28

Free-Body Diagram. As shown on the free-body diagram of the control volume (elbow) Fig. 15–28*b*, the *fixed* connection at A exerts a resultant couple moment $\mathbf{M}_O$ and force components $\mathbf{F}_x$ and $\mathbf{F}_y$ on the elbow. Due to the static pressure of water in the pipe, the pressure force acting on the open control surface at A is $F_A = p_A A_A$. Since $1\text{ kPa} = 1000\text{ N/m}^2$,

$$F_A = p_A A_A = [100(10^3)\text{ N/m}^2][\pi(0.1\text{ m})^2] = 3141.6\text{ N}$$

There is no static pressure acting at B, since the water is discharged at atmospheric pressure; i.e., the pressure measured by a gauge at B is equal to zero, $p_B = 0$.

Equations of Steady Flow.

$$\overset{+}{\rightarrow}\Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax});\ -F_x + 3141.6\text{ N} = 200\text{ kg/s}(0 - 6.37\text{ m/s})$$

$$F_x = 4.41\text{ kN} \qquad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = \frac{dm}{dt}(v_{By} - v_{Ay});\ -F_y - 20(9.81)\text{ N} = 200\text{ kg/s}(-25.46\text{ m/s} - 0)$$

$$F_y = 4.90\text{ kN} \qquad \textit{Ans.}$$

If moments are summed about point O, Fig. 15–28*b*, then $\mathbf{F}_x$, $\mathbf{F}_y$, and the static pressure $\mathbf{F}_A$ are eliminated, as well as the moment of momentum of the water entering at A, Fig. 15–28*a*. Hence,

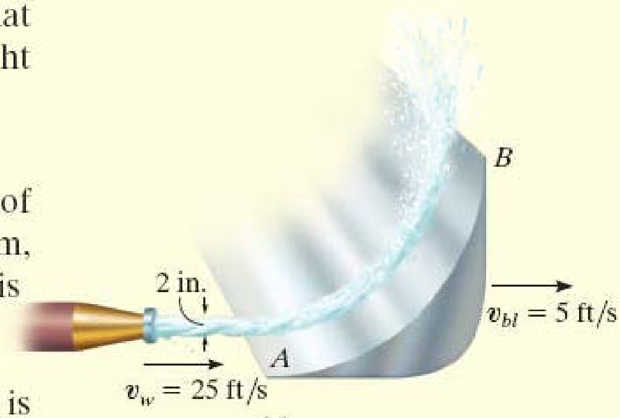
$$\circlearrowleft+\Sigma M_O = \frac{dm}{dt}(d_{OB}v_B - d_{OA}v_A)$$

$$M_O + 20(9.81)\text{ N }(0.125\text{ m}) = 200\text{ kg/s}[(0.3\text{ m})(25.46\text{ m/s}) - 0]$$

$$M_O = 1.50\text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

EXAMPLE 15.17

A 2-in.-diameter water jet having a velocity of 25 ft/s impinges upon a single moving blade, Fig. 15–29*a*. If the blade moves with a constant velocity of 5 ft/s away from the jet, determine the horizontal and vertical components of force which the blade is exerting on the water. What power does the water generate on the blade? Water has a specific weight of $\gamma_w = 62.4 \text{ lb/ft}^3$.



(a)

SOLUTION

Kinematic Diagram. Here the control volume will be the stream of water on the blade. From a fixed inertial coordinate system, Fig. 15–29*b*, the rate at which water enters the control volume at *A* is

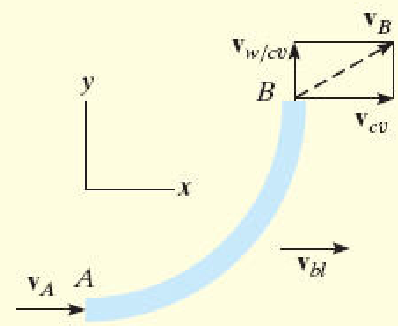
$$\mathbf{v}_A = \{25\mathbf{i}\} \text{ ft/s}$$

The *relative-flow velocity* within the control volume is $\mathbf{v}_{w/cv} = \mathbf{v}_w - \mathbf{v}_{cv} = 25\mathbf{i} - 5\mathbf{i} = \{20\mathbf{i}\}$ ft/s. Since the control volume is moving with a velocity of $\mathbf{v}_{cv} = \{5\mathbf{i}\}$ ft/s, the velocity of flow at *B* measured from the fixed *x*, *y* axes is the vector sum, shown in Fig. 15–29*b*. Here,

$$\mathbf{v}_B = \mathbf{v}_{cv} + \mathbf{v}_{w/cv}$$
$$= \{5\mathbf{i} + 20\mathbf{j}\} \text{ ft/s}$$

Thus, the mass flow of water *onto* the control volume that undergoes a momentum change is

$$\frac{dm}{dt} = \rho_w(v_{w/cv})A_A = \left(\frac{62.4}{32.2}\right)(20)\left[\pi\left(\frac{1}{12}\right)^2\right] = 0.8456 \text{ slug/s}$$



(b)

EXAMPLE 15.17 CONTINUED

Free-Body Diagram. The free-body diagram of the control volume is shown in Fig. 15–29*c*. The weight of the water will be neglected in the calculation, since this force will be small compared to the reactive components $\mathbf{F}_x$ and $\mathbf{F}_y$.

Equations of Steady Flow.

$$\Sigma \mathbf{F} = \frac{dm}{dt}(\mathbf{v}_B - \mathbf{v}_A)$$

$$-F_x\mathbf{i} + F_y\mathbf{j} = 0.8456(5\mathbf{i} + 20\mathbf{j} - 25\mathbf{i})$$

Equating the respective **i** and **j** components gives

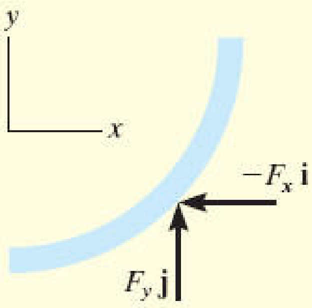
$$F_x = 0.8456(20) = 16.9 \text{ lb} \leftarrow \qquad \textit{Ans.}$$

$$F_y = 0.8456(20) = 16.9 \text{ lb} \uparrow \qquad \textit{Ans.}$$

The water exerts equal but opposite forces on the blade.

Since the water force which causes the blade to move forward horizontally with a velocity of 5 ft/s is $F_x = 16.9$ lb, then from Eq. 14–10 the power is

$$P = \mathbf{F} \cdot \mathbf{v}; \qquad P = \frac{16.9 \text{ lb}(5 \text{ ft/s})}{550 \text{ hp/(ft} \cdot \text{lb/s)}} = 0.154 \text{ hp}$$



(c)

Fig. 15–29

EXAMPLE 15.18

The initial combined mass of a rocket and its fuel is m_0. A total mass m_f of fuel is consumed at a constant rate of $dm_e/dt = c$ and expelled at a constant speed of u relative to the rocket. Determine the maximum velocity of the rocket, i.e., at the instant the fuel runs out. Neglect the change in the rocket's weight with altitude and the drag resistance of the air. The rocket is fired vertically from rest.

SOLUTION

Since the rocket loses mass as it moves upward, Eq. 15–28 can be used for the solution. The only *external force* acting on the *control volume* consisting of the rocket and a portion of the expelled mass is the weight **W**, Fig. 15–33. Hence,

$$+\uparrow \Sigma F_{cv} = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad -W = m\frac{dv}{dt} - uc \tag{1}$$

EXAMPLE 15.18 CONTINUED

The rocket's velocity is obtained by integrating this equation.

At any given instant t during the flight, the mass of the rocket can be expressed as $m = m_0 - (dm_e/dt)t = m_0 - ct$. Since $W = mg$, Eq. 1 becomes

$$-(m_0 - ct)g = (m_0 - ct)\frac{dv}{dt} - uc$$

Separating the variables and integrating, realizing that $v = 0$ at $t = 0$, we have

$$\int_0^v dv = \int_0^t \left(\frac{uc}{m_0 - ct} - g\right) dt$$

$$v = -u\ln(m_0 - ct) - gt\Big|_0^t = u\ln\left(\frac{m_0}{m_0 - ct}\right) - gt \quad (2)$$

Note that liftoff requires the first term on the right to be greater than the second during the initial phase of motion. The time t' needed to consume all the fuel is

$$m_f = \left(\frac{dm_e}{dt}\right)t' = ct'$$

Hence,

$$t' = m_f/c$$

Substituting into Eq. 2 yields

$$v_{max} = u\ln\left(\frac{m_0}{m_0 - m_f}\right) - \frac{gm_f}{c} \qquad \textit{Ans.}$$

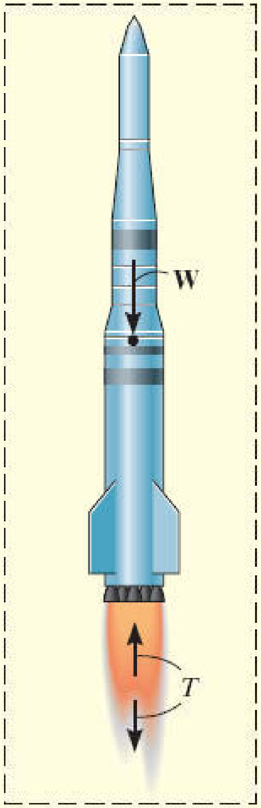
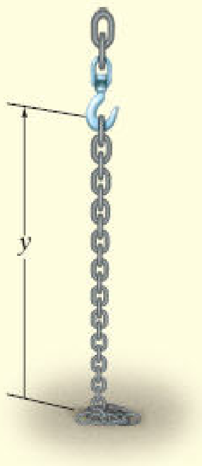
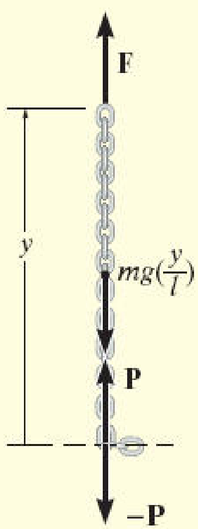


Fig. 15–33

EXAMPLE 15.19



(a)



(b)

A chain of length l, Fig. 15–34a, has a mass m. Determine the magnitude of force **F** required to (a) raise the chain with a constant speed v_c, starting from rest when $y = 0$; and (b) lower the chain with a constant speed v_c, starting from rest when $y = l$.

SOLUTION

Part (a). As the chain is raised, all the suspended links are given a sudden downward impulse by each added link which is lifted off the ground. Thus, the *suspended portion* of the chain may be considered as a device which is *gaining mass*. The control volume to be considered is the length of chain y which is suspended by **F** at any instant, including the next link which is about to be added but is still at rest, Fig. 15 34b. The forces acting on the control volume *exclude* the internal forces **P** and −**P**, which act between the added link and the suspended portion of the chain. Hence, $\Sigma F_{cv} = F - mg(y/l)$.

To apply Eq. 15–29, it is also necessary to find the rate at which mass is being added to the system. The velocity $\mathbf{v}_c$ of the chain is equivalent to $\mathbf{v}_{D/i}$. Why? Since v_c is constant, $dv_c/dt = 0$ and $dy/dt = v_c$. Integrating, using the initial condition that $y = 0$ when $t = 0$, gives $y = v_c t$. Thus, the mass of the control volume at any instant is $m_{cv} = m(y/l) = m(v_c t/l)$, and therefore the *rate* at which mass is *added* to the suspended chain is

$$\frac{dm_i}{dt} = m\left(\frac{v_c}{l}\right)$$

EXAMPLE 15.19 CONTINUED

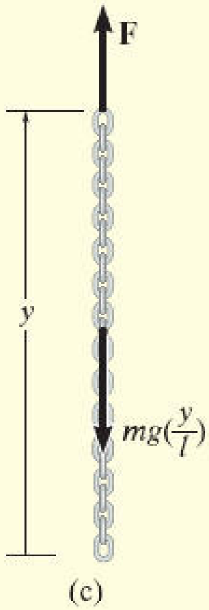


Fig. 15–34

Applying Eq. 15–29 using this data, we have

$$+\uparrow \Sigma F_{cv} = m\frac{dv_c}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$F - mg\left(\frac{y}{l}\right) = 0 + v_c m\left(\frac{v_c}{l}\right)$$

Hence,

$$F = (m/l)(gy + v_c^2) \qquad \textit{Ans.}$$

Part (b). When the chain is being lowered, the links which are expelled (given zero velocity) *do not* impart an impulse to the *remaining* suspended links. Why? Thus, the control volume in Part (*a*) will not be considered. Instead, the equation of motion will be used to obtain the solution. At time *t* the portion of chain still off the floor is *y*. The free-body diagram for a suspended portion of the chain is shown in Fig. 15–34*c*. Thus,

$$+\uparrow \Sigma F = ma; \qquad F - mg\left(\frac{y}{l}\right) = 0$$

$$F = mg\left(\frac{y}{l}\right) \qquad \textit{Ans.}$$