Carnot Cycle Quiz Solution

1. A Carnot cycle is conducted using air contained in a cylinder-piston configuration as shown in the figure below. Initially, the system contains air at 25 deg C, 100 kPa and 0.01 m^3. You may designate this as state 1. The system is first compressed isothermally until the volume is 0.002 m^3. From that point, the system undergoes a polytropic compression process with exponent n = 1.4 until the volume further reduces to 0.001 m^3. After that the system undergoes an isothermal expansion process to a point where pressure in the system is 500 kPa. At that point, the system is subjected to an isothermal expansion process until system reaches the initial state completing a cycle.

The maximum pressure in the cycle (in kPa) is:

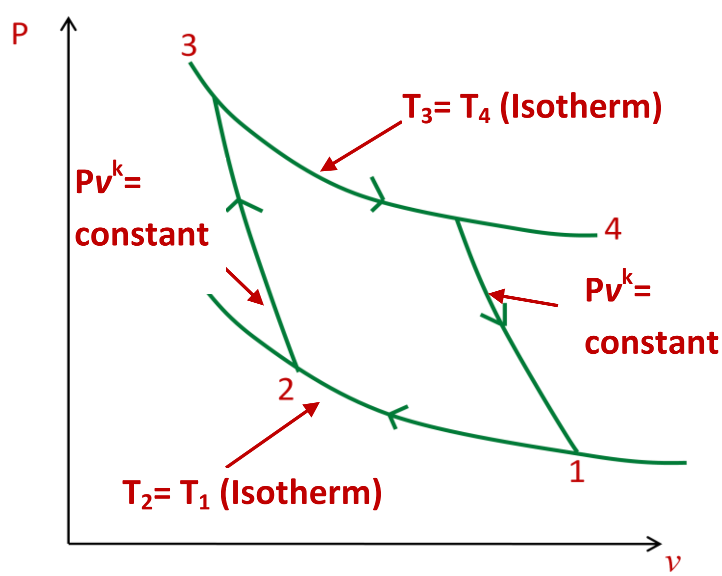
A. 1200

B. 1300

C. 1400

D. 1500

1. Solution



P_1 = 100 kPa, T_1 = 25 °C,

V_1 = 0.01 m^3,

The process 1→2 is an isothermal process.

$T_1 = T_2$ = 25 °C

V_1 = 0.002 m^3

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$

$$P_2 = \frac{P_1V_1}{V_2} = \frac{100 \times 0.01}{0.002}$$

$$= 500\ kPa$$

The process 2→3 is a polytropic process.

$V_1 = 0.001\ m^3$

$$P_3 V_3^{\,n} = P_2 V_2^{\,n}\ (n = k)$$

$$P_3 = P_2\left(\frac{V_2}{V_3}\right)^k = 500 * \left(\frac{0.002}{0.001}\right)^{1.4} = 1319\ kPa$$

Answer: B

2. A carnot cycle is conducted using air contained in a cylinder-piston configuration as shown in the figure below. Initially, the system contains air at 25 deg C, 100 kPa and 0.01 m³. You may designate this as state 1. The system is first compressed isothermally until the volume is 0.002 m³. From that point, the system undergoes a polytropic compression process with exponent n = 1.4 until the volume further reduces to 0.001 m³. After that the system undergoes an isothermal expansion process to a point where pressure in the system is 500 kPa. At that point, the system is subjected to an isothermal expansion process until system reaches the initial state completing a cycle.

The maximum temperature in the cycle (in K) is:

A. 390

B. 490

C. 590

D. 690

2. Solution

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$$

$$T_3 = \left(\frac{P_3}{P_2}\right)\left(\frac{V_3}{V_2}\right)T_2 = \frac{298.15 \times 1319 \times 0.001}{500 \times 0.002}$$

$$T_3 = 393.25\ K$$

Answer: A

3. A carnot cycle is conducted using air contained in a cylinder-piston configuration as shown in the figure below. Initially, the system contains air at 25 deg C, 100 kPa and 0.01 m^3. You may designate this as state 1. The system is first compressed isothermally until the volume is 0.002 m^3. From that point, the system undergoes a polytropic compression process with exponent n = 1.4 until the volume further reduces to 0.001 m^3. After that the system undergoes an isothermal expansion process to a point where pressure in the system is 500 kPa. At that point, the system is subjected to an isothermal expansion process until system reaches the initial state completing a cycle.

The volume after completion of the isothermal expansion process (in m^3) is:

A. 0.002

B. 0.003

C. 0.004

D. 0.005

3. Solution

During the isothermal expansion process,

$$\frac{P_3V_3}{T_3} = \frac{P_4V_4}{T_4}\ ;\ P_4 = 263.8\ kPa$$

$$V_4 = \frac{P_3V_3}{P_4} = \frac{1319 \times 0.001}{263.8} = 0.005\ \text{m}^3$$

Answer: D

4. A carnot cycle is conducted using air contained in a cylinder-piston configuration as shown in the figure below. Initially, the system contains air at 25 deg C, 100 kPa and 0.01 m^3. You may designate this as state 1. The system is first compressed isothermally until the volume is 0.002 m^3. From that point, the system undergoes a polytropic compression process with exponent n = 1.4 until the volume further reduces to 0.001 m^3. After that the system undergoes an isothermal expansion process to a point where pressure in the system is 500 kPa. At that point, the system is subjected to an isothermal expansion process until system reaches the initial state completing a cycle.

The magnitude of work done during the isothermal compression (in kJ) is:

A. 1.6

B. 2.6

C. 3.6

D. 4.6

4. Solution

$$_1W_2 = \int_{V_1}^{V_2} PdV$$

During the process 1→2,

$$PV = P_1V_1 = 100 \times 0.01\text{m}^3 = 1$$

$$_1W_2 = \int_{V_1}^{V_2} \frac{dV}{V} = ln\frac{V_2}{V_1}$$

$$_1W_2 = ln\frac{0.002}{0.01} = -1.609\ kJ$$

Magnitude of work = $\left|\ {}_1W_2\right| = 1.609\ kJ$

Answer: A

5. A carnot cycle is conducted using air contained in a cylinder-piston configuration as shown in the figure below. Initially, the system contains air at 25 deg C, 100 kPa and 0.01 m³. You may designate this as state 1. The system is first compressed isothermally until the volume is 0.002 m³. From that point, the system undergoes a polytropic compression process with exponent n = 1.4 until the volume further reduces to 0.001 m³. After that the system undergoes an isothermal expansion process to a point where pressure in the system is 500 kPa. At that point, the system is subjected to an isothermal expansion process until system reaches the initial state completing a cycle.

The magnitude of work of compression in the polytropic process (in KJ) is:

A. 0.6

B. 0.7

C. 0.8

D. 0.9

5. Solution

For the polytropic compression process (PV^k= constant),

$$ {}_2W_3 = \int_{V_2}^{V_3} PdV = \frac{P_3V_3 - P_2V_2}{1-k} $$

$$ {}_2W_3 = \frac{1319 \times 0.001 - 500 \times 0.002}{1 - 1.4} $$

$$ = -0.797\ kJ $$

Magnitude of work = $\left| {}_2W_3 \right| = 0.797\ kJ$

Answer: C

6. A carnot cycle is conducted using air contained in a cylinder-piston configuration as shown in the figure below. Initially, the system contains air at 25 deg C, 100 kPa and 0.01 m^3. You may designate this as state 1. The system is first compressed isothermally until the volume is 0.002 m^3. From that point, the system undergoes a polytropic compression process with exponent n = 1.4 until the volume further reduces to 0.001 m^3. After that the system undergoes an isothermal expansion process to a point where pressure in the system is 500 kPa. At that point, the system is subjected to an isothermal expansion process until system reaches the initial state completing a cycle.

The work during the isothermal expansion process (in kJ) is:

A. 1.1

B. 2.1

C. 3.1

D. 4.1

6. Solution

$$ {}_3W_4 = \int_{V_3}^{V_4} PdV $$

During the isothermal process 3→4,

$$ PV = C = P_3V_3 = 1319 \times 0.001 = 1.319 $$

$$ {}_3W_4 = \int_{V_3}^{V_4} \frac{1.319}{V} dV = 1.319\ ln\frac{V_4}{V_3} $$

$$ {}_3W_4 = 1.319\ ln\frac{0.005}{0.001} = \ 2.12\ kJ $$

Answer: B

7. A carnot cycle is conducted using air contained in a cylinder-piston configuration as shown in the figure below. Initially, the system contains air at 25 deg C, 100 kPa and 0.01 m^3. You may designate this as state 1. The system is first compressed isothermally until the volume is 0.002 m^3. From that point, the system undergoes a polytropic compression process with exponent n = 1.4 until the volume further reduces to 0.001 m^3. After that the system undergoes an isothermal expansion process to a point where pressure in the system is 500 kPa. At that point, the system is subjected to an isothermal expansion process until system reaches the initial state completing a cycle.

The work done during the polytropic expansion process (in kJ) is:

A. 0.5

B. 0.6

C. 0.7

D. 0.8

7. Solution

$$_4W_1 = \int_{V_4}^{V_1} PdV$$

For the polytropic process $PV^k = C$,

$$_4W_1 = \frac{P_1V_1 - P_4V_4}{1-k}$$

$$P_4 = 263.8\ kPa$$

$$V_4 = 0.005\mathrm{m}^3$$

$$_4W_1 = \frac{100 \times 0.01 - 0.005 \times 263.8}{1-1.4}$$

$$= 0.797\ kJ$$

Answer: D

8. A carnot cycle is conducted using air contained in a cylinder-piston configuration as shown in the figure below. Initially, the system contains air at 25 deg C, 100 kPa and 0.01 m^3. You may designate this as state 1. The system is first compressed isothermally until the volume is 0.002 m^3. From that point, the system undergoes a polytropic compression process with exponent n = 1.4 until the volume further reduces to 0.001 m^3. After that the system undergoes an isothermal expansion process to a point where pressure in the system is 500 kPa. At that point, the system is subjected to an isothermal expansion process until system reaches the initial state completing a cycle.

The net work of the cycle (in kJ) is:

A. 0.4

B. 0.5

C. 0.6

D. 0.7

8. Solution

$$W_{net} = {}_3W_4 + {}_4W_1 - |{}_1W_2| - |{}_2W_3|$$

$$W_{net} = 2.12 + 0.797 - 1.609 - 0.797$$

$$W_{net} = 0.511\ kJ$$

Answer: B

9. A carnot cycle is conducted using air contained in a cylinder-piston configuration as shown in the figure below. Initially, the system contains air at 25 deg C, 100 kPa and 0.01 m^3. You may designate this as state 1. The system is first compressed isothermally until the volume is 0.002 m^3. From that point, the system undergoes a polytropic compression process with exponent n = 1.4 until the volume further reduces to 0.001 m^3. After that the system undergoes an isothermal expansion process to a point where pressure in the system is 500 kPa. At that point, the system is subjected to an isothermal expansion process until system reaches the initial state completing a cycle.

The heat input during the isothermal expansion process (in kJ) is:

A. 2.1

B. 3.1

C. 4.1

D. 5.1

9. Solution

Heat input (Process 3→4)

First law: System

$${}_3Q_4 = U_4 - U_3 + {}_3W_4$$

$$= mC_{v0}(T_4 - T_3) + {}_3W_4$$

Since $T_4 = T_3$,

$${}_3Q_4 = {}_3W_4 = 2.12\ kJ$$

Answer: A

10. A carnot cycle is conducted using air contained in a cylinder-piston configuration as shown in the figure below. Initially, the system contains air at 25 deg C, 100 kPa and 0.01 m^3. You may designate this as state 1. The system is first compressed isothermally until the volume is 0.002 m^3. From that point, the system undergoes a polytropic compression process with exponent n = 1.4 until the volume further reduces to 0.001 m^3. After that the system undergoes an isothermal expansion process to a point where pressure in the system is 500 kPa. At that point, the system is subjected to an isothermal expansion process until system reaches the initial state completing a cycle.

The heat input during the polytropic expansion process (in kJ) is:

A. 0

B. 1

C. 2

D. 3

10. Solution

Polytropic expansion process 4→1,

$$ {}_4Q_1 = U_4 - U_1 + {}_4W_1 $$

$$ = mC_{v0}(T_1 - T_4) + {}_4W_1 $$

$$ T_1 = 298.15\ K;\ T_4 = T_3 = 393.15\ K $$

$$ m = \frac{P_1V_1}{RT_1} = \frac{100 \times 0.01}{0.287 \times 298.15} = 0.011686\ kg $$

$$ {}_4W_1 = 0.797\ kJ $$

$$ {}_4Q_1 = 0.011686 \times 0.717(298.15 - 393.15) + 0.797 $$

$$ {}_4Q_1 = -0.796 + 0.797 = 0.001\ kJ $$

Answer: A

11. A carnot cycle is conducted using air contained in a cylinder-piston configuration as shown in the figure below. Initially, the system contains air at 25 deg C, 100 kPa and 0.01 m^3. You may designate this as state 1. The system is first compressed isothermally until the volume is 0.002 m^3. From that point, the system undergoes a polytropic compression process with exponent n = 1.4 until the volume further reduces to 0.001 m^3. After that the system undergoes an isothermal expansion process to a point where pressure in the system is 500 kPa. At that point, the system is subjected to an isothermal expansion process until system reaches the initial state completing a cycle.

The heat rejected during the polytropic compression process (in kJ) is:

A. 3

B. 2

C. 1

D. 0

11. Solution

Process 3→4 (Polytropic Compression)

$${}_2Q_3 = U_3 - U_2 + {}_2W_3$$

$${}_2Q_3 = mC_{v0}(T_3 - T_2) + {}_2W_3$$

$${}_2W_3 = -0.797\ kJ$$

$${}_2Q_3 = 0.01168 \times 0.717(393.25 - 298.15) - 0.797$$

$${}_2Q_3 = 0.796 - 0.797 = -0.001\ kJ$$

Answer: D

12. A carnot cycle is conducted using air contained in a cylinder-piston configuration as shown in the figure below. Initially, the system contains air at 25 deg C, 100 kPa and 0.01 m^3. You may designate this as state 1. The system is first compressed isothermally until the volume is 0.002 m^3. From that point, the system undergoes a polytropic compression process with exponent n = 1.4 until the volume further reduces to 0.001 m^3. After that the system undergoes an isothermal expansion process to a point where pressure in the system is 500 kPa. At that point, the system is subjected to an isothermal expansion process until system reaches the initial state completing a cycle.

The cycle efficiency (in %) is:

A. 14

B. 24

C. 34

D. 44

12. Solution

$$n_{th} = \frac{W_{net}}{Q_H} = \frac{0.511}{2.12} = 0.24\ (24\%)$$

Also try

$$n_{th} = 1 - \frac{T_L}{T_H}\ (for\ Carnot\ Cycle)$$

$$n_{th} = 1 - \frac{298.15}{393.15} = 1 - 0.758$$

$$= 0.241\ (24\%)$$

Answer: B

13. A carnot cycle is conducted using air contained in a cylinder-piston configuration as shown in the figure below. Initially, the system contains air at 25 deg C, 100 kPa and 0.01 m^3. You may designate this as state 1. The system is first compressed isothermally until the volume is 0.002 m^3. From that point, the system undergoes a polytropic compression process with exponent n = 1.4 until the volume further reduces to 0.001 m^3. After that the system undergoes an isothermal expansion process to a point where pressure in the system is 500 kPa. At that point, the system is subjected to an isothermal expansion process until system reaches the initial state completing a cycle.

The mean effective pressure (in kPa) is:

A. 57

B. 67

C. 77

D. 87

13. Solution

$$mep = \frac{W_{net}}{(V_1 - V_3)} = \frac{0.511}{(0.01 - 0.001)}$$

$$= 56.77\ kPa$$

Answer: A