University Of Dundee

MATHEMATICS

An Introduction to Matlab

Version 2.1

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Contents

1 MATLAB

- Matlab is an interactive system for doing numerical computations.
- A numerical analyst called Cleve Moler wrote the first version of Matlab in the 1970s. It has since evolved into a successful commercial software package.
- Matlab relieves you of a lot of the mundane tasks associated with solving problems numerically. This allows you to spend more time thinking, and encourages you to experiment.
- Matlab makes use of highly respected algorithms and hence you can be confident about your results.
- Powerful operations can be performed using just one or two commands.
- You can build up your own set of functions for a particular application.
- Excellent graphics facilities are available, and the pictures can be inserted into LaTeX documents.

2 Starting Up

- You should have a directory reserved for saving files associated with Matlab. Create such a directory (`mkdir`) if you do not have one. Change into this directory (`cd`).
- Start up a new `xterm` window (do `xterm &` in the existing `xterm` window).
- Launch Matlab in one of the `xterm` windows with the command

  ```
  matlab
  ```

  After a short pause, the logo will be shown followed by

  where `>>` is the Matlab prompt.

  Type `help help` for "help" and `quit` **to exit from Matlab**.

3 Matlab as a Calculator

The basic arithmetic operators are `+ - * / ^` and these are used in conjunction with brackets: `( )`. The symbol `^` is used to get exponents (powers): `2^4=16`.
You should type in commands shown following the prompt: `>>`.

```
>> 2 + 3/4*5
ans =
    5.7500
>>
```

Is this calculation `2 + 3/(4*5)` or `2 + (3/4)*5`? Matlab works according to the priorities:

1. quantities in brackets,
2. powers `2 + 3^2` ⇒`2 + 9 = 11`,
3. `* /`, working left to right (`3*4/5=12/5`),
4. `+ -`, working left to right (`3+4-5=7-5`),

Thus, the earlier calculation was for `2 + (3/4)*5` by priority 3.

4 Numbers & Formats

Matlab recognizes several different kinds of numbers

Type	Examples
Integer	1362, −217897
Real	1.234, −10.76
Complex	$3.21 - 4.3i \quad (i = \sqrt{-1})$
Inf	Infinity (result of dividing by 0)
NaN	**N**ot **a** **N**umber, 0/0

The "`e`" notation is used for very large or very small numbers:
`-1.3412e+03` $= -1.3412 \times 10^3 = -1341.2$
`-1.3412e-01` $= -1.3412 \times 10^{-1} = -0.13412$
All computations in MATLAB are done in double precision, which means about 15 significant figures. The format—how Matlab prints numbers—is controlled by the "format" command. Type `help format` for full list.

Command	Example of Output
`>>format short`	`31.4162`(4–decimal places)
`>>format short e`	`3.1416e+01`
`>>format long e`	`3.141592653589793e+01`
`>>format short`	`31.4162`(4–decimal places)
`>>format bank`	`31.42`(2–decimal places)

Should you wish to switch back to the default format then `format` will suffice.
The command

```
format compact
```

is also useful in that it supresses blank lines in the output thus allowing more information to be displayed.

5 Variables

```
>> 3-2^4
ans =
   -13
>> ans*5
ans =
   -65
```

The result of the first calculation is labelled "`ans`" by Matlab and is used in the second calculation where its value is changed.
We can use our own names to store numbers:

```
>> x = 3-2^4
x =
   -13
>> y = x*5
y =
   -65
```

so that `x` has the value -13 and $y = -65$. These can be used in subsequent calculations. These are examples of **assignment statements**: values are assigned to variables. Each variable must be assigned a value before it may be used on the right of an assignment statement.

5.1 Variable Names

Legal names consist of any combination of letters and digits, starting with a letter. These are allowable:

```
NetCost, Left2Pay, x3, X3, z25c5
```

These are **not** allowable:

```
Net-Cost, 2pay, %x, @sign
```

Use names that reflect the values they represent.
Special names: you should avoid using
`eps = 2.2204e-16` $= 2^{-54}$ (The largest number such that `1 + eps` is indistinguishable from 1) and `pi = 3.14159...` $= \pi$.
If you wish to do arithmetic with complex numbers,both `i` and `j` have the value $\sqrt{-1}$ unless you change them

```
>> i,j, i=3
ans = 0 + 1.0000i
ans = 0 + 1.0000i
i   = 3
```

6 Suppressing output

One often does not want to see the result of intermediate calculations—terminate the assignment statement or expression with semi–colon

```
>> x=-13; y = 5*x, z = x^2+y
y =
   -65
z =
   104
>>
```

the value of `x` is hidden. Note also we can place several statements on one line, separated by commas or semi–colons.

Exercise 6.1 *In each case find the value of the expression in Matlab and explain precisely the order in which the calculation was performed.*

i)	`-2^3+9`	ii)	`2/3*3`
iii)	`3*2/3`	iv)	`3*4-5^2*2-3`
v)	`(2/3^2*5)*(3-4^3)^2`	vi)	`3*(3*4-2*5^2-3)`

7 Built–In Functions

7.1 Trigonometric Functions

Those known to Matlab are
`sin, cos, tan`
and their arguments should be in radians.
e.g. to work out the coordinates of a point on a circle of radius 5 centred at the origin and having an elevation $30^o = \pi/6$ radians:

```
>> x = 5*cos(pi/6), y = 5*sin(pi/6)
x =
    4.3301
y =
    2.5000
```

The inverse trig functions are called `asin, acos, atan` (as opposed to the usual arcsin or $\sin^{-1}$ etc.). The result is in radians.

```
>> acos(x/5), asin(y/5)
ans = 0.5236
ans = 0.5236
>> pi/6
ans = 0.5236
```

7.2 Other Elementary Functions

These include `sqrt, exp, log, log10`

```
>> x = 9;
>> sqrt(x),exp(x),log(sqrt(x)),log10(x^2+6)
ans =
     3
ans =
   8.1031e+03
ans =
    1.0986
ans =
    1.9395
```

`exp(x)` denotes the exponential function $\exp(x) = e^x$ and the inverse function is `log`:

```
>> format long e, exp(log(9)), log(exp(9))
ans = 9.000000000000002e+00
ans = 9
>> format short
```

and we see a tiny rounding error in the first calculation. `log10` gives logs to the base 10. A more complete list of elementary functions is given in Table 1 on page 26.

8 Vectors

These come in two flavours and we shall first describe **row vectors**: they are lists of numbers separated by either commas or spaces. The number of entries is known as the "length" of the vector and the entries are often referred to as "elements" or "components" of the vector.The entries must be enclosed in square brackets.

```
>> v = [ 1 3, sqrt(5)]
v =
    1.0000    3.0000    2.2361
>> length(v)
ans =
     3
```

Spaces can be vitally important:

```
>> v2 = [3+ 4 5]
v2 =
     7     5
>> v3 = [3 +4 5]
v3 =
     3     4     5
```

We can do certain arithmetic operations with vectors of the same length, such as `v` and `v3` in the previous section.

```
>> v + v3
ans =
    4.0000    7.0000    7.2361
>> v4 = 3*v
v4 =
    3.0000    9.0000    6.7082
>> v5 = 2*v -3*v3
v5 =
   -7.0000   -6.0000  -10.5279
>> v + v2
??? Error using ==> +
Matrix dimensions must agree.
```

i.e. the error is due to `v` and `v2` having different lengths.

A vector may be multiplied by a scalar (a number—see `v4` above), or added/subtracted to another vector of the **same** length. The operations are carried out elementwise.
We can build row vectors from existing ones:

```
>> w  = [1 2 3],  z = [8 9]
>> cd = [2*z,-w], sort(cd)
w =
     1     2     3
z =
     8     9
cd =
    16    18    -1    -2    -3
ans =
    -3    -2    -1    16    18
```

Notice the last command `sort`'ed the elements of `cd` into ascending order.
We can also change or look at the value of particular entries

```
>> w(2) = -2, w(3)
w =
     1    -2     3
ans =
     3
```

8.1 The Colon Notation

This is a shortcut for producing row vectors:

```
>>  1:4
ans =
     1     2     3     4
>> 3:7
ans =
     3     4     5     6     7
>> 1:-1
ans =
     []
```

More generally $a : b : c$ produces a vector of entries starting with the value a, incrementing by the value b until it gets to c (it will not produce a value beyond c). This is why `1:-1` produced the empty vector `[]`.

```
>> 0.32:0.1:0.6
ans =
    0.3200    0.4200    0.5200
>> -1.4:-0.3:-2
ans =
   -1.4000   -1.7000   -2.0000
```

8.2 Extracting Bits of a Vector

```
>>  r5 = [1:2:6, -1:-2:-7]
r5 =
     1     3     5    -1    -3    -5    -7
```

To get the 3rd to 6th entries:

```
>> r5(3:6)
ans =
     5    -1    -3    -5
```

To get alternate entries:

```
>> r5(1:2:7)
ans =
     5    -1    -3    -5
```

What does `r5(6:-2:1)` give?
See `help colon` for a fuller description.

8.3 Column Vectors

These have similar constructs to row vectors. When defining them, entries are separated by `;` or "newlines"

```
>> c = [ 1; 3; sqrt(5)]
c =
    1.0000
    3.0000
    2.2361
>> c2 = [3
4
5]
c2 =
     3
     4
     5
>> c3 = 2*c - 3*c2
c3 =
   -7.0000
   -6.0000
  -10.5279
```

so column vectors may be added or subtracted **provided that they have the same length**.

8.4 Transposing

We can convert a row vector into a column vector (and vice versa) by a process called *transposing*—denoted by `'`.

```
>> w, w', c, c'
w =
     1    -2     3
ans =
     1
    -2
     3
c =
    1.0000
    3.0000
    2.2361
ans =
    1.0000    3.0000    2.2361
>> t = w + 2*c'
t =
    3.0000    4.0000    7.4721
>> T = 5*w'-2*c
T =
    3.0000
  -16.0000
   10.5279
```

If `x` is a *complex vector*, then `x'` gives the *complex conjugate transpose* of `x`:

```
>> x = [1+3i, 2-2i]
ans =
   1.0000 + 3.0000i   2.0000 - 2.0000i
>> x'
ans =
   1.0000 - 3.0000i
   2.0000 + 2.0000i
```

Note that the components of `x` were defined without a `*` operator; this means of defining complex numbers works even when the variable `i` already has a numeric value. To obtain the plain transpose of a complex number use `.'` as in

```
>> x.'
ans =
   1.0000 + 3.0000i
   2.0000 - 2.0000i
```

9 Keeping a record

Issuing the command

```
>> diary mysession
```

will cause all subsequent text that appears on the screen to be saved to the file `mysession` located in the directory in which Matlab was invoked. You may use any legal filename *except* the names `on` and `off`. The record may be terminated by

```
>> diary off
```

The file `mysession` may be edited with emacs to remove any mistakes.
If you wish to quit Matlab midway through a calculation so as to continue at a later stage:

```
>> save thissession
```

will save the current values of all variables to a file called `thissession.mat`. **This file cannot be edited**. When you next startup Matlab, type

```
>> load thissession
```

and the computation can be resumed where you left off.
A list of variables used in the current session may be seen with

```
>> whos
```

See `help whos` and `help save`.

```
>> whos
Name  Size Elements  Bytes  Density Complex
ans   1 by 1  1        8      Full      No
  v   1 by 3  3       24      Full      No
 v1   1 by 2  2       16      Full      No
 v2   1 by 2  2       16      Full      No
 v3   1 by 3  3       24      Full      No
 v4   1 by 3  3       24      Full      No
  x   1 by 1  1        8      Full      No
  y   1 by 1  1        8      Full      No

Grand total is 16 elements using 128 bytes
```

10 Plotting Elementary Functions

Suppose we wish to plot a graph of $y = \sin 3\pi x$ for $0 \le x \le 1$. We do this by sampling the function at a sufficiently large number of points and then joining up the points (x, y) by straight lines. Suppose we take $N+1$ points equally spaced a distance h apart:

```
>> N = 10; h = 1/N; x = 0:h:1;
```

defines the set of points $x = 0, h, 2h, \ldots, 1-h, 1$. The corresponding y values are computed by

```
>> y = sin(3*pi*x);
```

and finally, we can plot the points with

```
>> plot(x,y)
```

The result is shown in Figure 1, where it is clear that the value of N is too small.

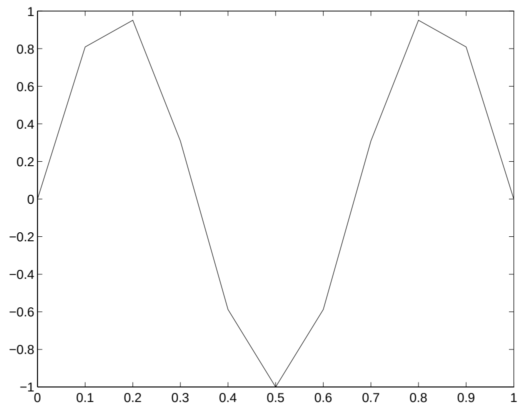


Figure 1: Graph of $y = \sin 3\pi x$ for $0 \le x \le 1$ using $h = 0.1$.

On changing the value of N to 100:

```
>> N = 100; h = 1/N; x = 0:h:1;
>> y = sin(3*pi*x);  plot(x,y)
```

we get the picture shown in Figure 2.

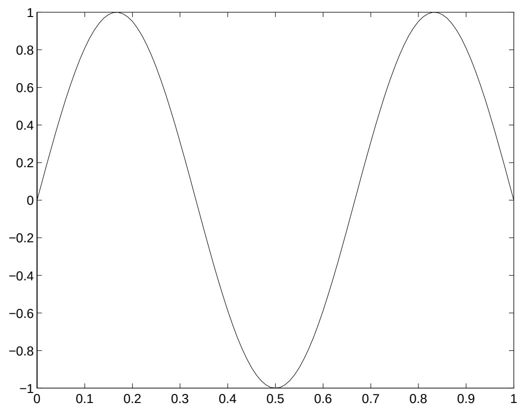


Figure 2: Graph of $y = \sin 3\pi x$ for $0 \le x \le 1$ using $h = 0.01$.

10.1 Plotting—Titles & Labels

To put a title and label the axes, we use

```
>> title('Graph of y = sin(3pi x)')
>> xlabel('x axis')
>> ylabel('y-axis')
```

The *strings* enclosed in single quotes, can be anything of our choosing (it is not straightforward to get formatted mathematical expressions as in LaTeX).

10.2 Grids

A dotted grid may be added by

```
>> grid
```

This can be removed using either `grid` again, or `grid off`.

10.3 Line Styles & Colours

The default is to plot solid lines. A solid white line is produced by

```
>> plot(x,y,'w-')
```

The third argument is a string whose first character specifies the colour(optional) and the second the line style. The options for colours and styles are:

Colours		Line Styles	
`y`	yellow	`.`	point
`m`	magenta	`o`	circle
`c`	cyan	`x`	x-mark
`r`	red	`+`	plus
`g`	green	`-`	solid
`b`	blue	`*`	star
`w`	white	`:`	dotted
`k`	black	`-.`	dashdot
		`--`	dashed

10.4 Multi–plots

Several graphs may be drawn on the same figure as in

```
>> plot(x,y,'w-',x,cos(2*pi*x),'g--')
```

A descriptive legend may be included with

```
>> legend('Sin curve','Cos curve')
```

which will give a list of line–styles, as they appeared in the plot command, followed by a brief description. Matlab fits the legend in a suitable position, so as not to conceal the graphs whenever possible. For further information do `help plot` etc.
The result of the commands

```
>> plot(x,y,'w-',x,cos(2*pi*x),'g--')
>> legend('Sin curve','Cos curve')
>> title('Multi-plot ')
>> xlabel('x axis'),  ylabel('y axis')
>> grid
```

is shown in Figure 3. The legend may be moved manually by dragging it with the mouse.

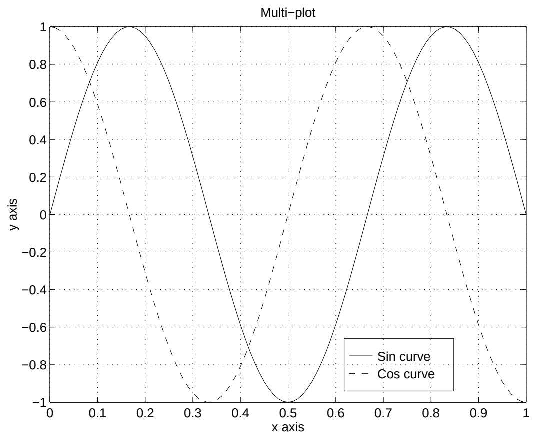


Figure 3: Graph of $y = \sin 3\pi x$ and $y = \cos 3\pi x$ for $0 \le x \le 1$ using $h = 0.01$.

10.5 Hold

A call to `plot` clears the graphics window before plotting the current graph. This is not convenient if we wish to add further graphics to the figure at some later stage. To stop the window being cleared:

```
>> plot(x,y,'w-'), hold
>> plot(x,y,'gx'), hold off
```

"`hold on`" holds the current picture; "`hold off`" releases it (but does not clear the window, which can be done with `clg`). "`hold`" on its own toggles the hold state.

10.6 Hard Copy

To obtain a printed copy on the bubblejet printer:

1. Issue the Matlab command

   ```
   print -deps fig1
   ```

   which will save a copy of the image in a file called `fig1.eps` (Encapsulated PostScript).

2. Move the mouse pointer into another `xterm` window, check that it is looking at the same directory (`pwd`) and issue the **Unix** command

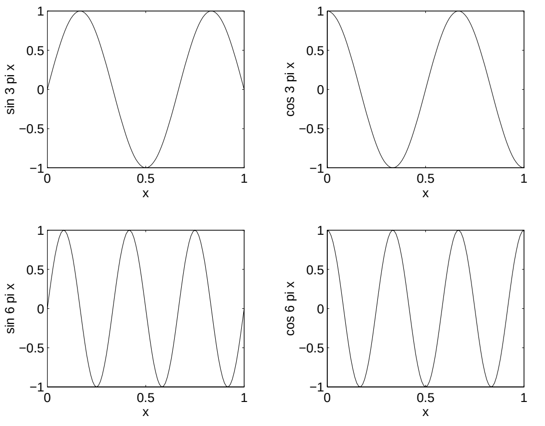
   ```
   lpr -Pbj  fig1.eps
   ```

10.7 Subplot

The graphics window may be split into an $m \times n$ array of smaller windows into which we may plot one or more graphs. The windows are counted 1 to mn row–wise, starting from the top left. Both `hold` and `grid` work on the current subplot.

```
>> subplot(221), plot(x,y)
>>   xlabel('x'),ylabel('sin 3 pi x')
>> subplot(222), plot(x,cos(3*pi*x))
>>   xlabel('x'),ylabel('cos 3 pi x')
>> subplot(223), plot(x,sin(6*pi*x))
>>   xlabel('x'),ylabel('sin 6 pi x')
>> subplot(224), plot(x,cos(6*pi*x))
>>   xlabel('x'),ylabel('cos 6 pi x')
```

`subplot(221)` (or `subplot(2,2,1)`) specifies that the window should be split into a 2×2 array and we select the first subwindow.



10.8 Zooming

We often need to "zoom in" on some portion of a plot in order to see more detail. This is easily achieved using the command

```
>> zoom
```

Pointing the mouse to the relevant position on the plot and clicking the left mouse button will zoom in by a factor of two. This may be repeated to any desired level.
Clicking the right mouse button will zoom out by a factor of two.
Holding down the left mouse button and dragging the mouse will cause a rectangle to be outlined. Releasing the button causes the contents of the rectangle to fill the window.
`zoom off` turns off the zoom capability.

Exercise 10.1 *Draw graphs of the functions*

$$\begin{aligned} y &= \cos x \\ y &= x \end{aligned}$$

for $0 \leq x \leq 2$ on the same window. Use the zoom facility to determine the point of intersection of the two curves (and, hence, the root of $x = \cos x$) to two significant figures.

10.9 Controlling Axes

Once a plot has been created in the graphics window you may wish to change the range of x and y values shown on the picture.

```
>> clg, N = 100; h = 1/N; x = 0:h:1;
>> y = sin(3*pi*x); plot(x,y)
>> axis([-0.5 1.5 -1.2 1.2]),  grid
```

The `axis` command has four parameters, the first two are the minimum and maximum values of x to use on the axis and the last two are the minimum and maximum values of y. Note the square brackets. The result of these commands is shown in Figure 4. Look at `help axis` and experiment with the commands `axis('equal')`, `axis('auto')`, `axis('square')`, `axis('normal')`, in any order.

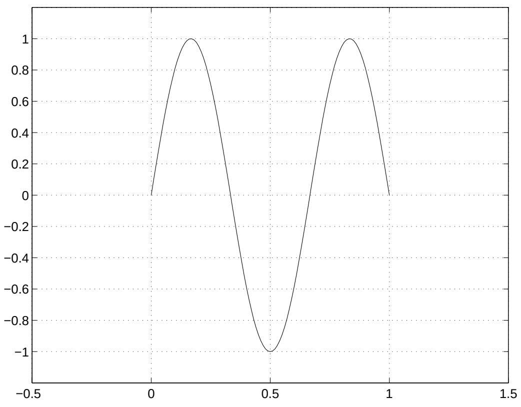


Figure 4: The effect of changing the axes of a plot.

11 Keyboard Accelerators

One can recall previous Matlab commands by using the ↑ and ↓ cursor keys. Repeatedly pressing ↑ will review the previous commands (most recent first) and, if you want to re-execute the command, simply press the return key.
To recall the most recent command starting with `p`, say, type `p` at the prompt followed by ↑. Similarly, typing `pr` followed by ↑ will recall the most recent command starting with `pr`.
Once a command has been recalled, it may be edited (changed). You can use ← and → to move backwards and forwards through the line, characters may be inserted by typing at the current cursor position or deleted using the `Del` key. This is most commonly used when long command lines have been mistyped or when you want to re–execute a command that is very similar to one used previously.
The following emacs commands may also be used:

`cntrl a`	move to start of line
`cntrl e`	move to end of line
`cntrl f`	move forwards one character
`cntrl b`	move backwards one character
`cntrl d`	delete character under the cursor

Once you have the command in the required form, press return.

Exercise 11.1 *Type in the commands*

```
>> x = -1:0.1:1;
>> plot(x,sin(pi*x),'w-')
>> hold on
>> plot(x,cos(pi*x),'r-')
```

Now use the cursor keys with suitable editing to execute:

```
>> x = -1:0.05:1;
>> plot(x,sin(2*pi*x),'w-')
>> plot(x,cos(2*pi*x),'r-.'), hold off
```

12 Copying to and from Emacs

There are many situations where one wants to copy the output resulting from a Matlab command (or commands) into a file being edited in Emacs. The rules are the same as for copying text in an Emacs window.
In order to carry out the following exercise, you should have Matlab running in one window and Emacs running in another.
To copy material from Matlab into Emacs: (L means click Left Mouse Button, etc)

Select the material to copy: L on the start of the material you want in the Matlab window, R at the end then move the mouse into the Emacs window and L at the location you want the text to appear. Finally, click the M.
The process for copying commands from an emacs file into Matlab is entirely similar, except that you can only copy material to the prompt line. You may copy as many lines as you want.

Exercise 12.1 1. *Copy the file*

~`dfg/NAP/Matlab/CopyExercise.m`

into your own area:

~`dfg/NAP/Matlab/CopyExercise.m` .

Load the file into emacs. You should also have Matlab running in another `xterm` *window.*

2. *Copy the commands one at a time from the file into Matlab.*
3. *Type* `CopyExercise` *at the Matlab prompt—you should see the results of the commands being executed.*
4. *Type* `echo on` *at the Matlab prompt and then* `CopyExercise`*—you should see the commands as well as the results.* `echo off` *will switch off echoing.*

13 Script Files

The last part of Exercise 12.1 introduced the idea of a *script* file. This is a normal ASCII (text) file that contains Matlab commands. It is essential that the file name should have an extension `.m` (e.g. `Exercise4.m`) and, for this reason, they are commonly known as *m-files*. The commands in this file may then be executed using

```
>> Exercise4
```

Note: this command does not include the file name extension `.m`.
It is only the output from the commands (and not the commands themselves) that are displayed on the screen. To see the commands:

```
>> echo on
```

and `echo off` will turn echoing off.
Any text that follows % on a line is ignored. The main purpose of this facility is to enable comments to be included in the file to describe its purpose.
To see what m-files you have in your current directory, use

```
>> what
```

Exercise 13.1 1. *Type in the commands from §10.7 into a file called* `exsub.m`*.*

2. *Use* `what` *to check that the file is in the correct area.*
3. *Use the command* `type exsub` *to see the contents of the file.*
4. *Execute these commands.*

See §20 for the related topic of function files.

14 Products, Division & Powers of Vectors

14.1 Scalar Product (*)

We shall describe two ways in which a meaning may be attributed to the product of two vectors. In both cases the vectors concerned must have the same length.
The first product is the standard scalar product. Suppose that $\underline{u}$ and $\underline{v}$ are two vectors of length n, $\underline{u}$ being a **row** vector and $\underline{v}$ a **column** vector:

$$\underline{u} = [u_1, u_2, \ldots, u_n], \qquad \underline{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}.$$

The scalar product is defined by multiplying the corresponding elements together and adding the results to give a single number (scalar).

$$\underline{u}\,\underline{v} = \sum_{i=1}^{n} u_i v_i.$$

For example, if $\underline{u} = [10, -11, 12]$, and $\underline{v} = \begin{bmatrix} 20 \\ -21 \\ -22 \end{bmatrix}$ then $n = 3$ and

$$\underline{u}\,\underline{v} = 10 \times 20 + (-11) \times (-21) + 12 \times (-22) = 167.$$

We can perform this product in Matlab by

```
>> u = [ 10, -11, 12], v = [20; -21; -22]
>> prod = u*v      % row times column vector
```

Suppose we also define a row vector `w` and a column vector `z` by

```
>> w = [2, 1, 3], z = [7; 6; 5]
w =
     2     1     3
z =
     7
     6
     5
```

and we wish to form the scalar products of `u` with `w` and `v` with `z`.

```
>> u*w
??? Error using ==> *
Inner matrix dimensions must agree.
```

an error results because `w` is not a column vector. Recall from page 5 that transposing (with `'`) turns column vectors into row vectors and vice versa.

So, to form the scalar product of two row vectors or two column vectors,

```
>> u*w'       % u & w are row vectors
ans =
    45
>> u*u'       % u is a row vector
ans =
   365
>> v'*z       % v & z are column vectors
ans =
   -96
```

We shall refer to the Euclidean length of a vector as the **norm** of a vector; it is denoted by the symbol $\|\underline{u}\|$ and defined by

$$\|\underline{u}\| = \sqrt{\sum_{i=1}^{n} |u_i|^2},$$

where n is its dimension. This can be computed in Matlab in one of two ways:

```
>> [ sqrt(u*u'), norm(u)]
ans =
   19.1050    19.1050
```

where `norm` is a built–in Matlab function that accepts a vector as input and delivers a scalar as output. It can also be used to compute other norms: `help norm`.

Exercise 14.1 *The angle, θ, between two column vectors $\underline{x}$ and $\underline{y}$ is defined by*

$$\cos\theta = \frac{\underline{x}'\underline{y}}{\|\underline{x}\|\,\|\underline{y}\|}.$$

Use this formula to determine the cosine of the angle between

$$\underline{x} = [1,2,3]' \quad \textit{and} \quad \underline{y} = [3,2,1]'.$$

Hence find the angle in degrees.

14.2 Dot Product (.*)

The second way of forming the product of two vectors of the same length is known as the Hadamard product. It is not often used in Mathematics but is an invaluable Matlab feature. It involves vectors of the same type. If $\underline{u}$ and $\underline{v}$ are two vectors of the same type (both row vectors or both column vectors), the mathematical definition of this product, which we shall call the **dot product**, is the **vector** having the components

$$\underline{u}\cdot\underline{v} = [u_1v_1, u_2v_2, \ldots, u_nv_n].$$

The result is a vector of the same length and type as $\underline{u}$ and $\underline{v}$. Thus, we simply multiply the corresponding elements of two vectors.
In Matlab, the product is computed with the operator `.*` and, using the vectors `u, v, w, z` defined on page 9,

```
>> u.*w
ans =
      20 -11 36
>> u.*v'
ans =
      200 231 -264
>> v.*z, u'.*v
ans =
      140 -126 -110
ans =
      200 231 -264
```

Example 14.1 *Tabulate the function $y = x\sin\pi x$ for $x = 0, 0.25, \ldots, 1$.*

It is easier to deal with column vectors so we first define a vector of x-values: (see Transposing: §8.4)
`>> x = (0:0.25:1)';`
To evaluate y we have to multiply each element of the vector x by the corresponding element of the vector $\sin\pi x$:

$x\ \times$	$\sin\pi x =$	$x\sin\pi x$
0 ×	0 =	0
0.2500 ×	0.7071 =	0.1768
0.5000 ×	1.0000 =	0.5000
0.7500 ×	0.7071 =	0.5303
1.0000 ×	0.0000 =	0.0000

To carry this out in Matlab:

```
>>  y = x.*sin(pi*x)
y =
         0
    0.1768
    0.5000
    0.5303
    0.0000
```

Note: a) the use of `pi`, b) `x` and `sin(pi*x)` are both column vectors (the `sin` function is applied to each element of the vector). Thus, the dot product of these is also a column vector.

14.3 Dot Division of Arrays (./)

There is no mathematical definition for the division of one vector by another. However, in Matlab, the operator `./` is defined to give element by element division—it is therefore only defined for vectors of the same size and type.

```
>> a = 1:5, b = 6:10, a./b
a =
     1     2     3     4     5
b =
     6     7     8     9    10
ans =
 0.1667  0.2857  0.3750  0.4444  0.5000
>> a./a
ans =
     1     1     1     1     1
>> c = -2:2, a./c
c =
    -2    -1     0     1     2
Warning: Divide by zero
ans =
 -0.5000  -2.0000  Inf   4.0000   2.5000
```

The previous calculation required division by 0—notice the `Inf`, denoting infinity, in the answer.

```
>> a.*b -24, ans./c
ans =
   -18   -10     0    12    26

Warning: Divide by zero
ans =
     9    10   NaN    12    13
```

Here we are warned about 0/0—giving a `NaN` (**N**ot **a** **N**umber).

Example 14.2 *Estimate the limit*

$$\lim_{x\to 0}\frac{\sin \pi x}{x}.$$

The idea is to observe the behaviour of the ratio $\frac{\sin \pi x}{x}$ for a sequence of values of x that approach zero. Suppose that we choose the sequence defined by the column vector
`>> x = [0.1; 0.01; 0.001; 0.0001]`
then

```
>> sin(pi*x)./x
ans =
    3.0902
    3.1411
    3.1416
    3.1416
```

which suggests that the values approach π. To get a better impression, we subtract the value of π from each entry in the output and, to display more decimal places, we change the format

```
>> format long
>> ans -pi
ans =
  -0.05142270984032
  -0.00051674577696
  -0.00000516771023
  -0.00000005167713
```

Can you explain the pattern revealed in these numbers?
We also need to use `./` to compute a scalar divided by a vector:

```
>> 1/x
??? Error using ==> /
Matrix dimensions must agree.
>> 1./x
ans =
   10    100    1000    10000
```

so `1./x` works, but `1/x` does not.

14.4 Dot Power of Arrays (.^)

To square each of the elements of a vector we could, for example, do `u.*u`. However, a neater way is to use the `.^` operator:

```
>> u.^2
ans =
   100   121   144
>> u.*u
ans =
   100   121   144
>> u.^4
ans =
       10000       14641       20736
>> v.^2
ans =
   400
   441
   484
>> u.*w.^(-2)
ans =
    2.5000  -11.0000    1.3333
```

Recall that powers (`.^` in this case) are done first, before any other arithmetic operation.

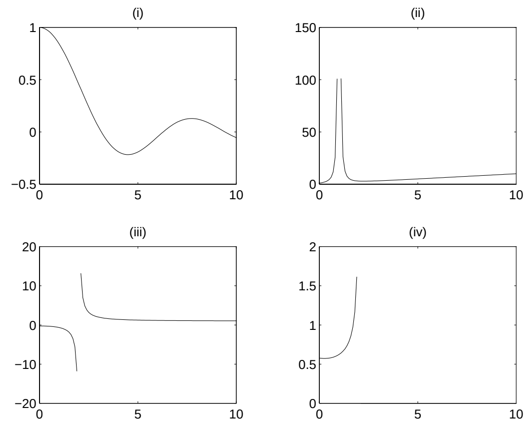
15 Examples in Plotting

Example 15.1 *Draw graphs of the functions*

i) $y = \frac{\sin x}{x}$ *ii)* $u = \frac{1}{(x-1)^2} + x$
iii) $v = \frac{x^2+1}{x^2-4}$ *iv)* $w = \frac{(10-x)^{1/3}-2}{(4-x^2)^{1/2}}$

for $0 \le x \le 10$.

```
>> x = 0:0.1:10;
>> y = sin(x)./x;
>>     subplot(221), plot(x,y), title('(i)')
Warning: Divide by zero
>> u = 1./(x-1).^2 + x;
>>     subplot(222),plot(x,u), title('(ii)')
Warning: Divide by zero
>> v = (x.^2+1)./(x.^2-4);
>>     subplot(223),plot(x,v),title('(iii)')
Warning: Divide by zero
>> w = ((10-x).^(1/3)-1)./sqrt(4-x.^2);
Warning: Divide by zero
>>     subplot(224),plot(x,w),title('(iv)')
```



Note the repeated use of the "dot" operators. Experiment with changing the axes (page 8), grids (page 6)and hold(page 7).

```
>>  subplot(222),axis([0 10 0 10])
>>  grid
>>  grid
>>  hold on
>>  plot(x,v,'--'), hold off, plot(x,y,':')
```

Exercise 15.1 *Enter the vectors*

$$\underline{U} = [6, 2, 4], \quad \underline{V} = [3, -2, 3, 0],$$
$$\underline{W} = \begin{bmatrix} 3 \\ -4 \\ 2 \\ -6 \end{bmatrix}, \quad \underline{Z} = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 7 \end{bmatrix}$$

into Matlab.

1. *Which of the products*

   `U*V,  V*W,  U*V', V*W', W*Z', U.*V`

   `U'*V, V'*W, W'*Z, U.*W, W.*Z, V.*W`

   is legal? State whether the legal products are row or column vectors and give the values of the legal results.

2. *Tabulate the functions*

$$y = (x^2 + 3)\sin \pi x^2$$

   and

$$z = \sin^2 \pi x/(x^{-2} + 3)$$

   for $x = 0, 0.2, \ldots, 10$. *Hence, tabulate the function*

$$w = \frac{(x^2 + 3)\sin \pi x^2 \sin^2 \pi x}{(x^{-2} + 3)}.$$

   Plot a graph of w *over the range* $0 \le x \le 10$.

16 Matrices—Two–Dimensional Arrays

Row and Column vectors are special cases of **matrices**.

An $m \times n$ matrix is a rectangular array of numbers having m rows and n columns. It is usual in a mathematical setting to include the matrix in either round or square brackets—we shall use square ones. For example, when $m = 2, n = 3$ we have a 2×3 matrix such as

$$A = \begin{bmatrix} 5 & 7 & 9 \\ 1 & -3 & -7 \end{bmatrix}$$

To enter such an matrix into Matlab we type it in row by row using the same syntax as for vectors:

```
>> A = [5 7 9
        1 -3 -7]
A =
     5     7     9
     1    -3    -7
```

Rows may be separated by semi-colons rather than a new line:

```
>> B = [-1 2 5; 9 0 5]
B =
    -1     2     5
     9     0     5
>> C = [0, 1; 3, -2; 4, 2]
C =
     0     1
     3    -2
     4     2
>> D = [1:5; 6:10; 11:2:20]
D =
     1     2     3     4     5
     6     7     8     9    10
    11    13    15    17    19
```

So `A` and `B` are 2×3 matrices, `C` is 3×2 and `D` is 3×5.

In this context, a row vector is a $1 \times n$ matrix and a column vector a $m \times 1$ matrix.

16.1 Size of a matrix

We can get the size (dimensions) of a matrix with the command `size`

```
>> size(A), size(x)
ans =
     2     3
ans =
     3     1
>> size(ans)
ans =
     1     2
```

So `A` is 2×3 and x is 3×1 (a column vector). The last command `size(ans)` shows that the *value* returned by `size` is itself a 1×2 matrix (a row vector). We can save the results for use in subsequent calculations.

```
>> [r c] = size(A'), S = size(A')
r =
     3
c =
     2
S =
     3     2
```

16.2 Transpose of a matrix

Transposing a vector changes it from a row to a column vector and vice versa (see §8.4). The extension of this idea to matrices is that transposing interchanges rows with the corresponding columns: the 1st row becomes the 1st column, and so on.

```
>> D, D'
D =
     1     2     3     4     5
     6     7     8     9    10
    11    13    15    17    19
ans =
     1     6    11
     2     7    13
     3     8    15
     4     9    17
     5    10    19
>> size(D), size(D')
ans =
     3     5
ans =
     5     3
```

16.3 Special Matrices

Matlab provides a number of useful built–in matrices of any desired size.
`ones(m,n)` gives an $m \times n$ matrix of 1's,

```
>> P = ones(2,3)
P =
     1     1     1
     1     1     1
```

`zeros(m,n)` gives an $m \times n$ matrix of 0's,

```
>> Z = zeros(2,3), zeros(size(P'))
Z =
     0     0     0
     0     0     0
ans =
     0     0
     0     0
     0     0
```

The second command illustrates how we can construct a matrix based on the size of an existing one. Try `ones(size(D))`.
An $n \times n$ matrix that has the same number of rows and columns and is called a **square** matrix.
A matrix is said to be **symmetric** if it is equal to its transpose (i.e. it is unchanged by transposition):

```
>> S = [2 -1 0; -1 2 -1; 0 -1 2],
S =
     2    -1     0
    -1     2    -1
     0    -1     2
>> St = S'
St =
     2    -1     0
    -1     2    -1
     0    -1     2
>> S-St
ans =
     0     0     0
     0     0     0
     0     0     0
```

16.4 The Identity Matrix

The $n \times n$ **identity** matrix is a matrix of zeros except for having ones along its leading diagonal (top left to bottom right). This is called `eye(n)` in Matlab (since mathematically it is usually denoted by I).

```
>> I = eye(3), x = [8; -4; 1], I*x
I =
     1     0     0
     0     1     0
     0     0     1
x =
     8
    -4
     1
ans =
     8
    -4
     1
```

Notice that multiplying the 3×1 vector `x` by the 3×3 identity `I` has no effect (it is like multiplying a number by 1).

16.5 Diagonal Matrices

A diagonal matrix is similar to the identity matrix except that its diagonal entries are not necessarily equal to 1.

$$D = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

is a 3×3 diagonal matrix. To construct this in Matlab, we could either type it in directly

```
>> D = [-3 0 0; 0 4 0; 0 0 2]
D =
    -3     0     0
     0     4     0
     0     0     2
```

but this becomes impractical when the dimension is large (e.g. a 100×100 diagonal matrix). We then use the `diag` function.We first define a vector `d`, say, containing the values of the diagonal entries (in order) then `diag(d)` gives the required matrix.

```
>>  d = [-3 4 2], D = diag(d)
d =
    -3     4     2
D =
    -3     0     0
     0     4     0
     0     0     2
```

On the other hand, if `A` is any matrix, the command `diag(A)` extracts its diagonal entries:

```
>> F = [0 1 8 7; 3 -2 -4 2; 4 2 1 1]
F =
     0     1     8     7
     3    -2    -4     2
     4     2     1     1
>> diag(F)
ans =
     0
    -2
     1
```

Notice that the matrix does not have to be square.

16.6 Building Matrices

It is often convenient to build large matrices from smaller ones:

```
>> C=[0 1; 3 -2; 4 2]; x=[8;-4;1];
>> G = [C x]
G =
     0     1     8
     3    -2    -4
     4     2     1
>> A, B, H = [A; B]
A =
     5     7     9
     1    -3    -7
B =
    -1     2     5
     9     0     5
ans =
     5     7     9
     1    -3    -7
    -1     2     5
     9     0     5
```

so we have added an extra column (`x`) to `C` in order to form `G` and have stacked `A` and `B` on top of each other to form `H`.

```
>> J = [1:4; 5:8; 9:12; 20 0 5 4]
J =
     1     2     3     4
     5     6     7     8
     9    10    11    12
    20     0     5     4

>> K = [ diag(1:4) J; J' zeros(4,4)]
K =
   1   0   0   0   1   2   3   4
   0   2   0   0   5   6   7   8
   0   0   3   0   9  10  11  12
   0   0   0   4  20   0   5   4
   1   5   9  20   0   0   0   0
   2   6  10   0   0   0   0   0
   3   7  11   5   0   0   0   0
   4   8  12   4   0   0   0   0
```

The command `spy(K)` will produce a graphical display of the location of the nonzero entries in `K` (it will also give a value for `nz`—the number of nonzero entries):

```
>> spy(K), grid
```

16.7 Tabulating Functions

This has been addressed in earlier sections but we are now in a position to produce a more suitable table format.

Example 16.1 *Tabulate the functions* $y = 4 \sin 3x$ *and* $u = 3 \sin 4x$ *for* $x = 0, 0.1, 0.2, \ldots, 0.5$.

```
>> x = 0:0.1:0.5;
>> y = 4*sin(3*x); u = 3*sin(4*x);
>> [ x' y' u']
ans =
         0         0         0
    0.1000    1.1821    1.1683
    0.2000    2.2586    2.1521
    0.3000    3.1333    2.7961
    0.4000    3.7282    2.9987
    0.5000    3.9900    2.7279
```

Note the use of transpose (`'`) to get column vectors. (we could replace the last command by `[x; y; u;]'`) We could also have done this more directly:

```
>>  x = (0:0.1:0.5)';
>> [x 4*sin(3*x) 3*sin(4*x)]
```

16.8 Extracting Bits of Matrices

We may extract sections from a matrix in much the same way as for a vector (page 4).
Each element of a matrix is indexed according to which row and column it belongs to. The entry in the ith row and jth column is denoted mathematically by $A_{i,j}$ and, in Matlab, by `A(i,j)`. So

```
>> J
J =
     1     2     3     4
     5     6     7     8
     9    10    11    12
    20     0     5     4
>> J(1,1)
ans =
     1
>> J(2,3)
ans =
     7
>> J(4,3)
ans =
     5
>> J(4,5)
???  Index exceeds matrix dimensions.
>> J(4,1) = J(1,1) + 6
J =
     1     2     3     4
     5     6     7     8
     9    10    11    12
     7     0     5     4
>> J(1,1) = J(1,1) - 3*J(1,2)
J =
    -5     2     3     4
     5     6     7     8
     9    10    11    12
     7     0     5     4
```

In the following examples we extract i) the 3rd column, ii) the 2nd and 3rd columns, iii) the 4th row, and iv) the "central" 2×2 matrix. See §8.1.

```
>> J(:,3)          % 3rd column
ans =
     3
     7
    11
     5
>> J(:,2:3)        % columns 2 to 3
ans =
     2     3
     6     7
    10    11
     0     5
>> J(4,:)          % 4th row
ans =
     7     0     5     4
>> J(2:3,2:3)      % rows 2 to 3 & cols 2 to 3
ans =
     6     7
    10    11
```

Thus, `:` on its own refers to the entire column or row depending on whether it is the first or the second index.

16.9 Dot product of matrices (`.*`)

The dot product works as for vectors: corresponding elements are multiplied together—so the matrices involved must have the same size.

```
>> A, B
A =
     5     7     9
     1    -3    -7
B =
    -1     2     5
     9     0     5
>> A.*B
ans =
    -5    14    45
     9     0   -35
>> A.*C
??? Error using ==> .*
Matrix dimensions must agree.
>> A.*C'
ans =
     0    21    36
     1     6   -14
```

16.10 Matrix–vector products

We turn next to the definition of the product of a matrix with a vector. This product is only defined for **column vectors** that have the same number of entries as the matrix has columns. So, if A is an $m \times n$ matrix and $\underline{x}$ is a column vector of length n, then the matrix–vector $A\underline{x}$ is legal.

An $m \times n$ matrix times an $n \times 1$ matrix $\Rightarrow$ a $m \times 1$ matrix.
We visualise A as being made up of m row vectors stacked on top of each other, then the product corresponds to taking the **scalar** product (See §14.1) of each row of A with the vector $\underline{x}$: The result is a column vector with m entries.

$$
\begin{aligned}
A\underline{x} &= \left[\begin{array}{c} \boxed{5 \quad 7 \quad 9} \\ \boxed{1 \quad -3 \quad -7} \end{array}\right] \left[\boxed{\begin{array}{r} 8 \\ -4 \\ 1 \end{array}}\right] \\
&= \left[\begin{array}{c} 5 \times 8 + 7 \times (-4) + 9 \times 1 \\ 1 \times 8 + (-3) \times (-4) + (-7) \times 1 \end{array}\right] \\
&= \left[\begin{array}{c} 21 \\ 13 \end{array}\right]
\end{aligned}
$$

It is somewhat easier in Matlab:

```
>> A = [5 7 9; 1 -3 -7]
A =
     5     7     9
     1    -3    -7
>> x = [8; -4; 1]
x =
     8
    -4
     1
>> A*x
ans =
    21
    13
```

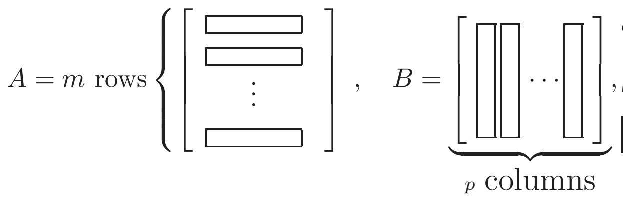
$$(m \times \boxed{n) \text{ times } (n} \times 1) \Rightarrow (m \times 1).$$

```
>> x*A
??? Error using ==> *
Inner matrix dimensions must agree.
```

Unlike multiplication in arithmetic, `A*x` **is not the same as** `x*A`.

16.11 Matrix–Matrix Products

To form the product of an $m \times n$ matrix A and a $n \times p$ matrix B, written as AB, we visualise the first matrix (A) as being composed of m row vectors of length n stacked on top of each other while the second (B) is visualised as being made up of p column vectors of length n:



The entry in the ith row and jth column of the product is then the **scalar**product of the ith row of A with the jth column of B. The product is an $m \times p$ matrix:

$$(m \times \boxed{n) \text{ times } (n} \times p) \Rightarrow (m \times p).$$

Check that you understand what is meant by working out the following examples by hand and comparing with the Matlab answers.

```
>> A = [5  7  9;  1 -3 -7]
A =
     5     7     9
     1    -3    -7
>> B = [0, 1; 3, -2; 4, 2]
B =
     0     1
     3    -2
     4     2
>> C = A*B
C =
    57     9
   -37    -7
>> D = B*A
D =
     1    -3    -7
    13    27    41
    22    22    22
>>  E = B'*A'
E =
    57   -37
     9    -7
```

We see that `E = C'` suggesting that

(A*B)' = B'*A'
Why is $B * A$ a 3×3 matrix while $A * B$ is 2×2?

17 Fireworks

As light relief, use your `xterm` window to copy three files to your area:

```
cp ~dfg/Matlab/fireworks.m  .
cp ~dfg/Matlab/comet*.m  .
```

then, in Matlab,

```
>> fireworks
```

A graphics window will pop up, in which you should click the left mouse button on $\boxed{\text{Fire}}$.

To end the demo, click on $\boxed{\text{Fire}}$ again, then on $\boxed{\text{Done}}$.

18 Loops

There are occasions that we want to repeat a segment of code a number of different times (such occasions are less frequent than other programming languages because of the : notation).

Example 18.1 *Draw graphs of* $\sin(n\pi x)$ *on the interval* $-1 \le x \le 1$ *for* $n = 1, 2, \ldots, 8$.

We could do this by giving 8 separate plot commands but it is much easier to use a loop. The simplest form would be

```
>> x = -1:.05:1;
>> for n = 1:8
     subplot(4,2,n), plot(x,sin(n*pi*x))
   end
```

All the commands between the lines starting "for" and "end" are repeated with n being given the value 1 the first time through, 2 the second time, and so forth, until $n = 8$. The subplot constructs a 4×2 array of subwindows and, on the nth time through the loop, a picture is drawn in the nth subwindow. The commands

```
>> x = -1:.05:1;
>> for n = 1:2:8
     subplot(4,2,n), plot(x,sin(n*pi*x))
     subplot(4,2,n+1), plot(x,cos(n*pi*x))
   end
```

draw $\sin n\pi x$ and $\cos n\pi x$ for $n = 1, 3, 5, 7$ alongside each other.
We may use any legal variable name as the "loop counter" (n in the above examples) and it can be made to run through all of the values in a given vector (1:8 and 1:2:8 in the examples).
We may also use for loops of the type

```
>> for counter = [23  11  19  5.4  6]
      .......
   end
```

which repeats the code as far as the end with counter=23 the first time, counter=11 the second time, and so forth.

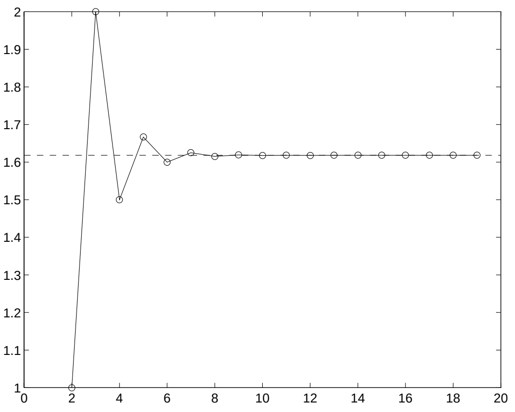
Example 18.2 *The Fibonnaci sequence starts off with the numbers* 0 *and* 1*, then succeeding terms are the sum of its two immediate predecessors. Mathematically,* $f_1 = 0$*,* $f_2 = 1$ *and*

$$f_n = f_{n-1} + f_{n-2}, \qquad n = 3, 4, 5, \ldots.$$

Test the assertion that the ratio f_n/f_{n-1} *of two successive values approaches the golden ratio* $(\sqrt{5}+1)/2$ $= 1.6180\ldots$.

```
>> F(1) = 0; F(2) = 1;
>> for i = 3:20
     F(i) = F(i-1) + F(i-2);
   end
>> plot(1:19, F(2:20)./F(1:19),'o' )
>> hold on
>> plot(1:19, F(2:20)./F(1:19),'-' )
>> plot([0 20], (sqrt(5)+1)/2*[1,1],'--')
```

The last of these commands produced the dashed horizontal line.



Example 18.3 *Produce a list of the values of the sums*

$$\begin{aligned} S_{20} &= 1 + \tfrac{1}{2^2} + \tfrac{1}{3^2} + \cdots + \tfrac{1}{20^2} \\ S_{21} &= 1 + \tfrac{1}{2^2} + \tfrac{1}{3^2} + \cdots + \tfrac{1}{20^2} + \tfrac{1}{21^2} \\ &\vdots \\ S_{100} &= 1 + \tfrac{1}{2^2} + \tfrac{1}{3^2} + \cdots + \tfrac{1}{20^2} + \tfrac{1}{21^2} + \cdots + \tfrac{1}{100^2} \end{aligned}$$

There are a total of 81 sums. The first can be computed using sum(1./(1:20).^2) (The function sum with a vector argument sums its components. See §21.2].) A suitable piece of Matlab code might be

```
>> S = zeros(100,1);
>> S(20) = sum(1./(1:20).^2);
>> for n = 21:100
>>   S(n) = S(n-1) + 1/n^2;
>> end
>> clg; plot(S,'.',[20 100],[1,1]*pi^2/6,'-')
>> axis([20 100 1.5 1.7])
>> [ (98:100)'  S(98:100)]
ans =
   98.0000    1.6364
   99.0000    1.6365
  100.0000    1.6366
```

where a column vector S was created to hold the answers. The first sum was computed directly using the sum command then each succeeding sum was found by adding $1/n^2$ to its predecessor. The little table at the end shows the values of the last three sums—it appears that they are approaching a limit (the value of the limit is $\pi^2/6 = 1.64493\ldots$).

Exercise 18.1 *Repeat Example 18.3 to include 181 sums (i.e. the final sum should include the term $1/200^2$.)*

19 Logicals

Matlab represents `true` and `false` by means of the integers 0 and 1.

`true = 1,    false = 0`

If at some point in a calculation a scalar `x`, say, has been assigned a value, we may make certain logical tests on it:

`x == 2`	is `x` equal to 2?
`x ~= 2`	is `x` **not** equal to 2?
`x > 2`	is `x` greater than 2?
`x < 2`	is `x` less than 2?
`x >= 2`	is `x` greater than or equal to 2?
`x <= 2`	is `x` less than or equal to 2?

Pay particular attention to the fact that the test for equality involves two equal signs `==`.

```
>> x = pi
x =
    3.1416
>> x ~= 3,  x ~= pi
ans =
     1
ans =
     0
```

When `x` is a vector or a matrix, these tests are performed elementwise:

```
x =
   -2.0000    3.1416    5.0000
   -1.0000         0    1.0000
>> x == 0
ans =
     0     0     0
     0     1     0
>> x > 1,  x >=-1
ans =
     0     1     1
     0     0     0
ans =
     0     1     1
     1     1     1
>> y = x>=-1, x > y
y =
     0     1     1
     1     1     1
ans =
     0     1     1
     0     0     0
```

We may combine logical tests, as in

```
>> x
x =
   -2.0000    3.1416    5.0000
   -5.0000   -3.0000   -1.0000
>> x > 3 & x < 4
ans =
     0     1     0
     0     0     0
>> x > 3 | x == -3
ans =
     0     1     1
     0     1     0
```

As one might expect, `&` represents **and** and (not so clearly) the vertical bar `|` means **or**; also `~` means **not** as in `~=` (not equal), `~(x>0)`, etc.

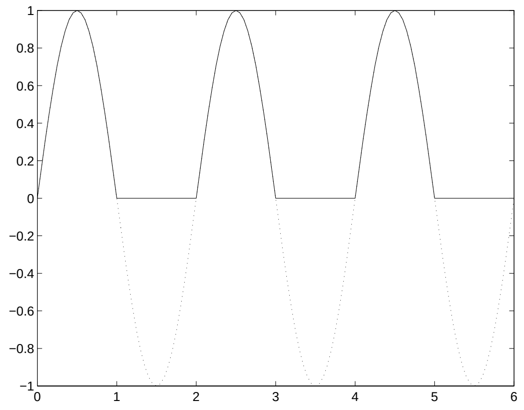
```
>> x > 3 | x == -3 | x <= -5
ans =
     0     1     1
     1     1     0
```

One of the uses of logical tests is to “mask out” certain elements of a matrix.

```
>> x, L =  x >= 0
x =
   -2.0000    3.1416    5.0000
   -5.0000   -3.0000   -1.0000
L =
     0     1     1
     0     1     1
>> pos = x.*L
pos =
         0    3.1416    5.0000
         0         0         0
```

so the matrix `pos` contains just those elements of `x` that are non–negative.

```
>> x = 0:0.05:6; y = sin(pi*x); Y = (y>=0).*y;
>> plot(x,y,':',x,Y,'-' )
```



19.1 While Loops

There are some occasions when we want to repeat a section of Matlab code until some logical condition is satisfied, but we cannot tell in advance how many times we have to go around the loop. This we can do with a `while...end` construct.

Example 19.1 *What is the greatest value of n that can be used in the sum*

$$1^2 + 2^2 + \cdots + n^2$$

and get a value of less than 100?

```
>> S = 1; n = 1;
>> while S+ (n+1)^2 < 100
     n = n+1;   S = S + n^2;
   end
>> [n, S]
ans =
     6    91
```

The lines of code between `while` and `end` will only be executed if the condition `S+ (n+1)^2 < 100` is true.

Exercise 19.1 *Replace 100 in the previous example by 10 and work through the lines of code by hand. You should get the answers $n = 2$ and $S = 5$.*

Exercise 19.2 *Type the code from Example19.1 into a script–file named `WhileSum.m` (See §13.)*

A more typical example is

Example 19.2 *Find the approximate value of the root of the equation $x = \cos x$. (See Example 10.1.)*

We may do this by making a guess $x_1 = \pi/4$, say, then computing the sequence of values

$$x_n = \cos x_{n-1}, \qquad n = 2, 3, 4, \ldots$$

and continuing until the difference between two successive values $|x_n - x_{n-1}|$ is small enough.

Method 1:

```
>> x = zeros(1,20); x(1) = pi/4;
>> n = 1;  d = 1;
>> while d > 0.001
     n = n+1; x(n) = cos(x(n-1));
     d = abs( x(n) - x(n-1) );
   end
   n,x
n =
    14
x =
Columns 1 through 7
0.7854 0.7071 0.7602 0.7247 0.7487 0.7326 0.7435
Columns 8 through 14
0.7361 0.7411 0.7377 0.7400 0.7385 0.7395 0.7388
Columns 15 through 20
   0    0    0    0    0    0
```

There are a number of deficiencies with this program. The vector `x` stores the results of each iteration but we don't know in advance how many there may be. In any event, we are rarely interested in the intermediate values of `x`, only the last one. Another problem is that we may never satisfy the condition $d \leq 0.001$, in which case the program will run forever—we should place a limit on the maximum number of iterations.
Incorporating these improvements leads to

Method 2:

```
>> xold = pi/4; n = 1;  d = 1;
>> while d > 0.001 &  n < 20
     n = n+1;  xnew = cos(xold);
     d = abs( xnew - xold );
     xold = xnew;
   end
>> [n, xnew, d]
ans =
   14.0000    0.7388    0.0007
```

We continue around the loop so long as $d > 0.001$ **and** $n < 20$. For greater precision we could use the condition $d > 0.0001$, and this gives

```
>> [n, xnew, d]
ans =
   19.0000    0.7391    0.0001
```

from which we may judge that the root required is $x = 0.739$ to 3 decimal places.
The general form of `while` statement is

```
while a logical test
          Commands to be executed
          when the condition is true
end
```

19.2 `if...then...else...end`

This allows us to execute different commands depending on the truth or falsity of some logical tests. To test whether or not π^e is greater than, or equal to, e^π:

```
>> a = pi^exp(1); c = exp(pi);
>> if a >= c
     b = sqrt(a^2 - c^2)
   end
```

so that `b` is assigned a value only if $a \geq c$. There is no output so we deduce that $a = \pi^e < c = e^\pi$. A more common situation is

```
>> if a >= c
     b = sqrt(a^2 - c^2)
   else
```

```
      b = 0
    end
b =
     0
```

which ensures that `b` is always assigned a value and confirming that $a < c$.
A more extended form is

```
>> if a >= c
     b = sqrt(a^2 - c^2)
   elseif a^c > c^a
     b = c^a/a^c
   else
     b = a^c/c^a
   end
b =
    0.2347
```

Exercise 19.3 *Which of the above statements assigned a value to* `b`*?*

The general form of the `if` statement is

```
if logical test 1
     Commands to be executed if test
     1 is true
elseif logical test 2
     Commands to be executed if test
     2 is true but test 1 is false
     ⋮

end
```

20 Function m–files

These are a combination of the ideas of script m–files (§7) and Mathematical functions.

Example 20.1 *The area, A, of a triangle with sides of length a, b and c is given by*

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where $s = (a+b+c)/2$. Write a Matlab function that will accept the values a, b and c as inputs and return the value os A as output.

The main steps to follow when defining a Matlab function are:

1. Decide on a name for the function, making sure that it does not conflict with a name that is already used by Matlab. In this example the name of the function is to be `area`, so its definition will be saved in a file called `area.m`
2. The first line of the file must have the format:

   `function` [*list of outputs*]

   = *function_name*(*list of inputs*)

   For our example, the output (A) is a function of the three variables (inputs) a, b and c so the first line should read

   ```
   function [A] = area(a,b,c)
   ```
3. Document the function. That is, describe briefly the purpose of the function and how it can be used. These lines should be preceded by `%` which signify that they are comment lines that will be ignored when the function is evaluated.
4. Finally include the code that defines the function. This should be interspersed with sufficient comments to enable another user to understand the processes involved.

The complete file might look like:

```
function  [A] = area(a,b,c)
% Compute the area of a triangle whose
% sides have length a, b and c.
% Inputs:
%    a,b,c:  Lengths of sides
% Output:
%        A: area of triangle
% Usage:
%      Area = area(2,3,4);
% Written by dfg, Oct 14, 1996.
s = (a+b+c)/2;
A = sqrt(s*(s-a)*(s-b)*(s-c));
%%%%%%%%% end of area %%%%%%%%%%%
```

The command
`>> help area`
will produce the leading comments from the file:

```
  Compute the area of a triangle whose
  sides have length a, b and c.
  Inputs:
     a,b,c:  Lengths of sides
  Output:
         A: area of triangle
  Usage:
       Area = area(2,3,4);
  Written by dfg, Oct 14, 1996.
```

To evalute the area of a triangle with side of length 10, 15, 20:

```
>> Area = area(10,15,20)
Area =
   72.6184
```

where the result of the computation is assigned to the variable `Area`. The variable `s` used in the definition of the function above is a "local variable": its value is local to the function and cannot be used outside:

```
>> s
??? Undefined function or variable s.
```

If we were to be interested in the value of s as well as A, then the first line of the file should be changed to

```
    function [A,s] = area(a,b,c)
```

where there are two output variables.
This function can be called in several different ways:

1. No outputs assigned

   ```
   >> area(10,15,20)
   ans =
      72.6184
   ```

   gives only the area (first of the output variables from the file) assigned to `ans`; the second output is ignored.

2. One output assigned

   ```
   >> Area = area(10,15,20)
   Area =
      72.6184
   ```

   again the second output is ignored.

3. Two outputs assigned

   ```
   >> [Area, hlen] = area(10,15,20)
   Area =
      72.6184
   hlen =
      22.5000
   ```

Exercise 20.1 *In any triangle the sum of the lengths of any two sides cannot exceed the length of the third side. The function* `area` *does not check to see if this condition is fulfilled (try* `area(1,2,4)`*). Modify the file so that it computes the area only if the sides satisfy this condition.*

20.1 Examples of functions

We revisit the problem of computing the Fibonnaci sequence defined by $f_1 = 0, f_2 = 1$ and

$$f_n = f_{n-1} + f_{n-2}, \qquad n = 3, 4, 5, \ldots.$$

We want to construct a function that will return the nth number in the Fibinnaci sequence f_n.

- **Input**: Integer n
- **Output**: f_n

We shall describe four possible functions and try to assess which provides the best solution.

Method 1: File `~dfg/Matlab/doc/Fib1.m`

```
function f = Fib1(n)
% Returns the nth number in the
% Fibonacci sequence.
F=zeros(1,n+1);
F(2) = 1;
   for i = 3:n+1
     F(i) = F(i-1) + F(i-2);
   end
f = F(n);
```

This code resembles that given in Example 18.2. We have simply enclosed it in a function m–file and given it the appropriate header,

Method 2: File `~dfg/Matlab/doc/Fib2.m`
The first version was rather wasteful of memory—it saved all the entries in the sequence even though we only required the last one for output. The second version removes the need to use a vector.

```
function f = Fib2(n)
% Returns the nth number in the
% Fibonacci sequence.
if n==1
   f = 0;
elseif n==2
   f = 1;
else
   f1 = 0; f2 = 1;
   for i = 2:n-1
     f = f1 + f2;
     f1=f2; f2 = f;
   end
end
```

Method 3: File: `~dfg/Matlab/doc/Fib3.m`
This version makes use of an idea called "recursive programming"— the function makes calls to itself.

```
function f = Fib3(n)
% Returns the nth number in the
% Fibonacci sequence.
if n==1
   f = 0;
elseif n==2
   f = 1;
else
   f = Fib3(n-1) + Fib3(n-2);
end
```

Method 4: File `~dfg/Matlab/doc/Fib4.m`
The final version uses matrix powers. The vector `y` has two components, $\underline{y} = \begin{bmatrix} f_n \\ f_{n+1} \end{bmatrix}$.

```
function f = Fib4(n)
% Returns the nth number in the
% Fibonacci sequence.
A = [0 1;1 1];
y = A^n*[1;0];
f=y(1);
```

Assessment: One may think that, on grounds of style, the 3rd is best (it avoids the use of loops) followed by the second (it avoids the use of a vector). The situation is much different when it cames to speed of execution. When $n = 20$ the time taken by each of the methods is (in seconds)

Method	Time
1	0.0118
2	0.0157
3	36.5937
4	0.0078

It is impractical to use Method 3 for any value of n much larger than 10 since the time taken by method 3 almost doubles whenever n is increased by just 1. When $n = 150$

Method	Time
1	0.0540
2	0.0891
3	—
4	0.0106

Clearly the 4th method is much the fastest.

21 Further Built–in Functions

21.1 Rounding Numbers

There are a variety of ways of rounding and chopping real numbers to give integers. Use the definitions given in the table in §26 on page 26 in order to understand the output given below:

```
>> x = pi*(-1:3),  round(x)
x =
  -3.1416  0    3.1416  6.2832   9.4248
ans =
    -3     0     3     6     9
>> fix(x)
ans =
    -3     0     3     6     9
>> floor(x)
ans =
    -4     0     3     6     9
>> ceil(x)
ans =
    -3     0     4     7    10
>> sign(x),  rem(x,3)
ans =
    -1     0     1     1     1
ans =
   -0.1416  0   0.1416   0.2832   0.4248
```

Do “`help round`” for help information.

21.2 The `sum` Function

The “`sum`” applied to a vector adds up its components (as in `sum(1:10)`) while, for a matrix, it adds up the components in **each column** and returns a row vector. `sum(sum(A))` then sums all the entries of `A`.

```
>>  A = [1:3; 4:6; 7:9]
A =
     1     2     3
     4     5     6
     7     8     9
>> s = sum(A), ss = sum(sum(A))
s =
    12    15    18
ss =
    45

>> x = pi/4*(1:3)';
>> A = [sin(x), sin(2*x), sin(3*x)]/sqrt(2)
>> A =
    0.5000    0.7071    0.5000
    0.7071    0.0000   -0.7071
    0.5000   -0.7071    0.5000

>> s1 = sum(A.^2), s2 = sum(sum(A.^2))
s1 =
    1.0000    1.0000    1.0000
s2 =
    3.0000
```

The sums of squares of the entries in each column of `A` are equal to 1 and the sum of squares of all the entries is equal to 3.

```
>> A*A'
ans =
    1.0000         0         0
         0    1.0000    0.0000
         0    0.0000    1.0000
>> A'*A
ans =
    1.0000         0         0
         0    1.0000    0.0000
         0    0.0000    1.0000
```

It appears that the products AA' and $A'A$ are both equal to the identity:

```
>> A*A' - eye(3)
ans =
   1.0e-15 *
   -0.2220         0         0
         0   -0.2220    0.0555
         0    0.0555   -0.2220
>> A'*A - eye(3)
ans =
   1.0e-15 *
   -0.2220         0         0
         0   -0.2220    0.0555
         0    0.0555   -0.2220
```

This is confirmed since the differences are at round–off error levels (less than 10^{-15}). A matrix with this property is called an *orthogonal* matrix.

21.3 max & min

These functions act in a similar way to `sum`. If `x` is a vector, then `max(x)` returns the largest element in `x`

```
>> x = [1.3 -2.4 0 2.3], max(x), max(abs(x))
x =
    1.3000   -2.4000         0    2.3000
ans =
    2.3000
ans =
    2.4000
>> [m, j] = max(x)
m =
    2.3000
j =
     4
```

When we ask for two outputs, the first gives us the maximum entry and the second the index of the maximum element.
For a matrix, `A`, `max(A)` returns a row vector containing the maximum element from each column. Thus to find the largest element in `A` we have to use `max(max(A))`.

21.4 Random Numbers

The function `rand(m,n)` produces an $m \times n$ matrix of random numbers, each of which is in the range 0 to 1. `rand` on its own produces a single random number.

```
>> y =  rand, Y = rand(2,3)
y =
    0.9191
Y =
    0.6262    0.1575    0.2520
    0.7446    0.7764    0.6121
```

Repeating these commands will lead to different answers.
Example: *Write a function–file that will simulate n throws of a pair of dice.*

This requires random numbers that are integers in the range 1 to 6. Multiplying each random number by 6 will give a real number in the range 0 to 6; rounding these to whole numbers will not be correct since it will then be possible to get 0 as an answer. We need to use

```
floor(1 + 6*rand)
```

Recall that `floor` takes the largest integer that is smaller than a given real number (see Table 1, page 26).
File: `~dfg/Matlab/doc/dice.m`

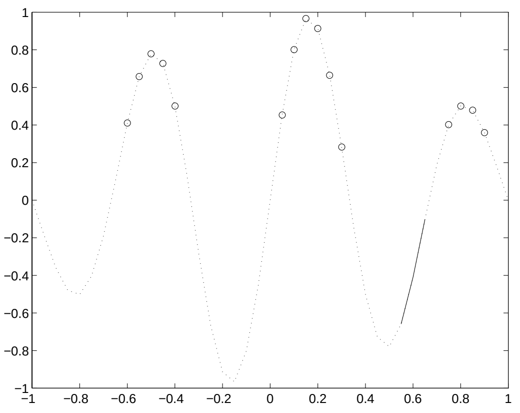
```
function  [d] = dice(n)
% simulates "n" throws of a pair of dice
% Input:     n, the number of throws
% Output:    an n times 2 matrix, each row
%            referring to one throw.
%
% Useage:  T = dice(3)
    d = floor(1 + 6*rand(n,2));
%% end of dice

>> dice(3)
ans =
     6     1
     2     3
     4     1
>> sum(dice(100))/100
ans =
    3.8500    3.4300
```

The last command gives the average value over 100 throws (it should have the value 3.5).

21.5 find for vectors

The function "`find`" returns a list of the positions (indices) of the elements of a vector satisfying a given condition. For example,

```
>> x = -1:.05:1;
>> y = sin(3*pi*x).*exp(-x.^2); plot(x,y,':')
>> k = find(y > 0.2)
 k =
  Columns 1 through 12
   9  10  11  12  13  22  23  24  25  26  27  36
  Columns 13 through 15
  37  38  39
>> hold on, plot(x(k),y(k),'o')
>> km = find( x>0.5 & y<0)
km =
    32    33    34
>> plot(x(km),y(km),'-')
```



21.6 find for matrices

The find–function operates in much the same way for matrices:

```
>> A = [ -2 3 4 4; 0 5 -1 6; 6 8 0 1]
A =
    -2     3     4     4
     0     5    -1     6
     6     8     0     1
>> k = find(A==0)
k =
     2
     9
```

Thus, we find that A has elements equal to 0 in positions 2 and 9. To interpret this result we have to recognise that "find" first reshapes A into a column vector—this is equivalent to numbering the elements of A by columns as in

$$\begin{array}{cccc} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 11 \\ 3 & 6 & 9 & 12 \end{array}$$

```
>> n  = find(A <= 0)
n =
     1
     2
     8
     9
>> A(n)
ans =
    -2
     0
    -1
     0
```

Thus, n gives a list of the locations of the entries in A that are ≤ 0 and then A(n) gives us the values of the elements selected.

```
>> m = find( A' == 0)
m =
     5
    11
```

Since we are dealing with A', the entries are numbered by rows.

22 Plotting Surfaces

A surface is defined mathematically by a function $f(x, y)$—corresponding to each value of (x, y) we compute the height of the function by

$$z = f(x, y).$$

In order to plot this we have to decide on the ranges of x and y—suppose $2 \leq x \leq 4$ and $1 \leq y \leq 3$. This gives us a square in the (x, y)–plane. Next, we need to choose a grid on this domain; Figure 5 shows the grid with intervals 0.5 in each direction. Finally, we

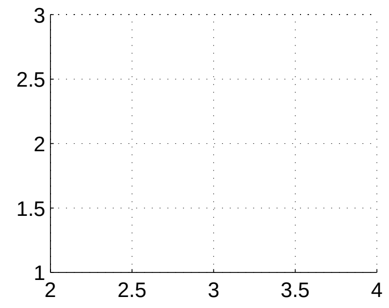


Figure 5: An example of a 2D grid

have to evaluate the function at each point of the grid and "plot" it.
Suppose we choose a grid with intervals 0.5 in each direction for illustration. The x– and y–coordinates of the grid lines are

```
x = 2:0.5:4;  y = 1:0.5:3;
```

in Matlab notation. We construct the grid with meshgrid:

```
>> [X,Y] = meshgrid(2:.5:4, 1:.5:3);
>> X
X =
    2.0000   2.5000   3.0000   3.5000   4.0000
    2.0000   2.5000   3.0000   3.5000   4.0000
    2.0000   2.5000   3.0000   3.5000   4.0000
    2.0000   2.5000   3.0000   3.5000   4.0000
    2.0000   2.5000   3.0000   3.5000   4.0000
>> Y
Y =
    1.0000   1.0000   1.0000   1.0000   1.0000
    1.5000   1.5000   1.5000   1.5000   1.5000
    2.0000   2.0000   2.0000   2.0000   2.0000
    2.5000   2.5000   2.5000   2.5000   2.5000
    3.0000   3.0000   3.0000   3.0000   3.0000
```

If we think of the ith point along from the left and the jth point up from the bottom of the grid) as corresponding to the (i, j)th entry in a matrix, then (X(i,j), Y(i,j)) are the coordinates of the point. We then need to evaluate the function f using X and Y in place of x and y, respectively.

Example 22.1 *Plot the surface defined by the function*

$$f(x,y) = (x-3)^2 - (y-2)^2$$

for $2 \leq x \leq 4$ *and* $1 \leq y \leq 3$.

```
>> [X,Y] = meshgrid(2:.2:4, 1:.2:3);
>> Z = (X-3).^2-(Y-2).^2;
>> mesh(X,Y,Z)
>> title('Saddle'), xlabel('x'),ylabel('y')
```

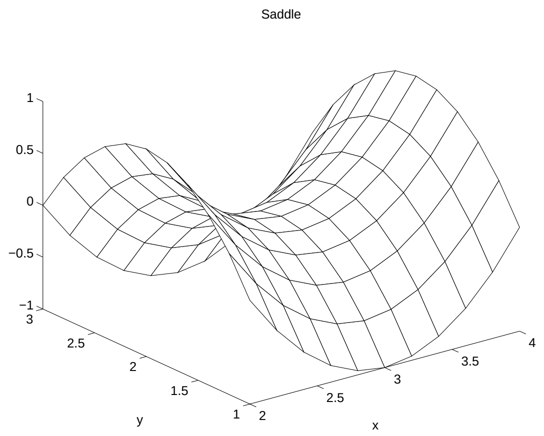


Figure 6: Plot of Saddle function.

Example 22.2 *Plot the surface defined by the function*

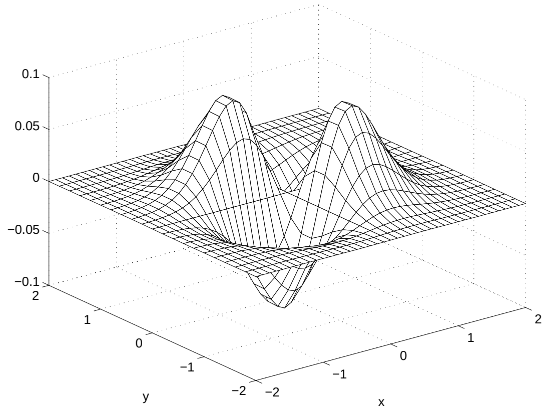
$$f = -xye^{-2(x^2+y^2)}$$

on the domain $-2 \leq x \leq 2, -2 \leq y \leq 2$. *Find the values and locations of the maxima and minima of the function.*

```
>> [X,Y] = meshgrid(-2:.1:2,-2:.2:2);
>> f =  -X.*Y.*exp(-2*(X.^2+Y.^2));
>> mesh(X,Y,f), xlabel('x'), ylabel('y'), grid
>> contour(X,Y,f)
>> xlabel('x'), ylabel('y'), grid, hold on
```

To locate the maxima of the "f" values on the grid:

```
>> fmax = max(max(f))
fmax =
    0.0886
>> kmax = find(f==fmax)
kmax =
   323
   539
>> Pos = [X(kmax), Y(kmax)]
Pos =
   -0.5000    0.6000
    0.5000   -0.6000
>> plot(X(kmax),Y(kmax),'*')
>> text(X(kmax),Y(kmax),' Maximum')
```



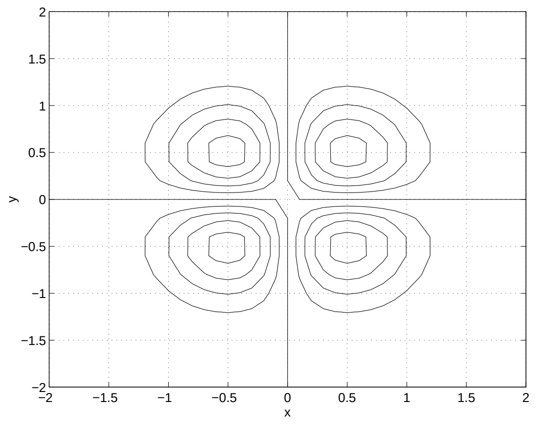


Figure 7: "mesh" and "contour" plots.

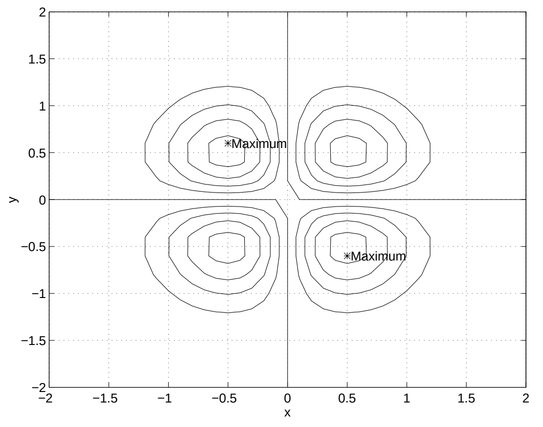


Figure 8: contour plot showing maxima.

23 Timing

Matlab allows the timing of sections of code by providing the functions tic and toc. tic switches on a stopwatch while toc stops it and returns the CPU time (Central Processor Unit) in seconds. The timings will vary depending on the model of computer being used and its current load.

```
>> tic,for j=1:1000,x = pi*R(3);end,toc
elapsed_time =     0.5110
>> tic,for j=1:1000,x=pi*R(3);end,toc
elapsed_time =    0.5017
```

```
>> tic,for j=1:1000,x=R(3)/pi;end,toc
elapsed_time =    0.5203
>> tic,for j=1:1000,x=pi+R(3);end,toc
elapsed_time =    0.5221
>> tic,for j=1:1000,x=pi-R(3);end,toc
elapsed_time =    0.5154
>> tic,for j=1:1000,x=pi^R(3);end,toc
elapsed_time =    0.6236
```

24 On–line Documentation

In addition to the on–line help facility, there is a hypertext browsing system giving details of (most) commands and some examples. This is accessed by

```
>> doc
```

which brings up the *Netscape* document previewer (and allows for "surfing the internet superhighway"—the World Wide Web (WWW). It is connected to a worldwide system which, given the appropriate addresses, will provide information on almost any topic).
Words that are underlined in the browser may be clicked on with LB and lead to either a further subindex or a help page.
Scroll down the page shown and click on general which will yake you to "General Purpose Commands"; click on clear. This will describe how you can clear a variable's value from memory.
You may then either click the "Table of Contents" which takes you back to the start, "Index" or the $\boxed{\text{Back}}$ button at the lower left corner of the window which will take you back to the previous screen.
To access other "home pages", click on $\boxed{\text{Open}}$ at the bottom of the window and, in the "box" that will open up, type

```
http://www.mcs.dundee.ac.uk
```

or

```
http://www.mcs.dundee.ac.uk/~dfg/homepage.html
```

25 Demos

Demonstrations are valuable since they give an indication of Matlabs capabilities.

```
>> demo
```

Warning: this will clear the values of all current variables. Click on $\boxed{\text{Continue}}$, then $\boxed{\text{Matlab/Visit}}$,

$\boxed{\text{Visualization/Select}}$, $\boxed{\text{XY plots}}$, etc.

26 Command Summary

The command
`>> help`
will give a list of categories for which help is available (e.g. `matlab/general` covers the topics listed in Table 2.

Further information regarding the commands listed in this section may then be obtained by using:
`>> help` *`topic`*
try, for example,
`>> help help`

`abs`	Absolute value
`sqrt`	Square root function
`sign`	Signum function
`conj`	Conjugate of a complex number
`imag`	Imaginary part of a complex number
`real`	Real part of a complex number
`angle`	Phase angle of a complex number
`cos`	Cosine function
`sin`	Sine function
`tan`	Tangent function
`exp`	Exponential function
`log`	Natural logarithm
`log10`	Logarithm base 10
`cosh`	Hyperbolic cosine function
`sinh`	Hyperbolic sine function
`tanh`	Hyperbolic tangent function
`acos`	Inverse cosine
`acosh`	Inverse hyperbolic cosine
`asin`	Inverse sine
`asinh`	Inverse hyperbolic sine
`atan`	Inverse tan
`atan2`	Two–argument form of inverse tan
`atanh`	Inverse hyperbolic tan
`round`	Round to nearest integer
`floor`	Round towards minus infinity
`fix`	Round towards zero
`ceil`	Round towards plus infinity
`rem`	Remainder after division

Table 1: Elementary Functions

Managing commands and functions.	
`help`	On-line documentation.
`doc`	Load hypertext documentation.
`what`	Directory listing of M-, MAT- and MEX-files.
`type`	List M-file.
`lookfor`	Keyword search through the HELP entries.
`which`	Locate functions and files.
`demo`	Run demos.
Managing variables and the workspace.	
`who`	List current variables.
`whos`	List current variables, long form.
`load`	Retrieve variables from disk.
`save`	Save workspace variables to disk.
`clear`	Clear variables and functions from memory.
`size`	Size of matrix.
`length`	Length of vector.
`disp`	Display matrix or text.
Working with files and the operating system.	
`cd`	Change current working directory.
`dir`	Directory listing.
`delete`	Delete file.
`!`	Execute operating system command.
`unix`	Execute operating system command & return result.
`diary`	Save text of MATLAB session.
Controlling the command window.	
`cedit`	Set command line edit/recall facility parameters.
`clc`	Clear command window.
`home`	Send cursor home.
`format`	Set output format.
`echo`	Echo commands inside script files.
`more`	Control paged output in command window.
Quitting from MATLAB.	
`quit`	Terminate MATLAB.

Table 2: General purpose commands.

Matrix analysis.	
`cond`	Matrix condition number.
`norm`	Matrix or vector norm.
`rcond`	LINPACK reciprocal condition estimator.
`rank`	Number of linearly independent rows or columns.
`det`	Determinant.
`trace`	Sum of diagonal elements.
`null`	Null space.
`orth`	Orthogonalization.
`rref`	Reduced row echelon form.
Linear equations.	
\ and `/`	Linear equation solution; use “help slash”.
`chol`	Cholesky factorization.
`lu`	Factors from Gaussian elimination.
`inv`	Matrix inverse.
`qr`	Orthogonal- triangular decomposition.
`qrdelete`	Delete a column from the QR factorization.
`qrinsert`	Insert a column in the QR factorization.
`nnls`	Non–negative least- squares.
`pinv`	Pseudoinverse.
`lscov`	Least squares in the presence of known covariance.
Eigenvalues and singular values.	
`eig`	Eigenvalues and eigenvectors.
`poly`	Characteristic polynomial.
`polyeig`	Polynomial eigenvalue problem.
`hess`	Hessenberg form.
`qz`	Generalized eigenvalues.
`rsf2csf`	Real block diagonal form to complex diagonal form.
`cdf2rdf`	Complex diagonal form to real block diagonal form.
`schur`	Schur decomposition.
`balance`	Diagonal scaling to improve eigenvalue accuracy.
`svd`	Singular value decomposition.
Matrix functions.	
`expm`	Matrix exponential.
`expm1`	M- file implementation of expm.
`expm2`	Matrix exponential via Taylor series.
`expm3`	Matrix exponential via eigenvalues and eigenvectors.
`logm`	Matrix logarithm.
`sqrtm`	Matrix square root.
`funm`	Evaluate general matrix function.

Table 3: Matrix functions—numerical linear algebra.

Graphics & plotting.	
`figure`	Create Figure (graph window).
`clf`	Clear current figure.
`close`	Close figure.
`subplot`	Create axes in tiled positions.
`axis`	Control axis scaling and appearance.
`hold`	Hold current graph.
`figure`	Create figure window.
`text`	Create text.
`print`	Save graph to file.
`plot`	Linear plot.
`loglog`	Log-log scale plot.
`semilogx`	Semi-log scale plot.
`semilogy`	Semi-log scale plot.
Specialized X-Y graphs.	
`polar`	Polar coordinate plot.
`bar`	Bar graph.
`stem`	Discrete sequence or "stem" plot.
`stairs`	Stairstep plot.
`errorbar`	Error bar plot.
`hist`	Histogram plot.
`rose`	Angle histogram plot.
`compass`	Compass plot.
`feather`	Feather plot.
`fplot`	Plot function.
`comet`	Comet-like trajectory.
Graph annotation.	
`title`	Graph title.
`xlabel`	X-axis label.
`ylabel`	Y-axis label.
`text`	Text annotation.
`gtext`	Mouse placement of text.
`grid`	Grid lines.
`contour`	Contour plot.
`mesh`	3-D mesh surface.
`surf`	3-D shaded surface.
`waterfall`	Waterfall plot.
`view`	3-D graph viewpoint specification.
`zlabel`	Z-axis label for 3-D plots.
`gtext`	Mouse placement of text.
`grid`	Grid lines.

Table 4: Graphics & plot commands.

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