Chapter 3, Solution 19.

The two surfaces of a window are maintained at specified temperatures. The rate of heat loss through the window and the inner surface temperature are to be determined.

Assumptions 1 Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. 2 Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. 3 Thermal conductivity is constant. 4 Heat transfer by radiation is negligible.

Properties The thermal conductivity of the glass is given to be \( k = 0.78 \text{ W/m} \cdot \text{°C} \).

Analysis The area of the window and the individual resistances are
\[
A = (1.2 \text{ m}) \times (2 \text{ m}) = 2.4 \text{ m}^2
\]

\[
R_1 = R_{\text{conv},1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot \text{°C})(2.4 \text{ m}^2)} = 0.04167 \text{ °C/W}
\]

\[
R_{\text{glass}} = \frac{L}{k_1 A} = \frac{0.006 \text{ m}}{(0.78 \text{ W/m} \cdot \text{°C})(2.4 \text{ m}^2)} = 0.00321 \text{ °C/W}
\]

\[
R_0 = R_{\text{conv},2} = \frac{1}{h_2 A} = \frac{1}{(25 \text{ W/m}^2 \cdot \text{°C})(2.4 \text{ m}^2)} = 0.01667 \text{ °C/W}
\]

\[
R_{\text{total}} = R_{\text{conv},1} + R_{\text{glass}} + R_{\text{conv},2}
= 0.04167 + 0.00321 + 0.01667 = 0.06155 \text{ °C/W}
\]

The steady rate of heat transfer through window glass is then
\[
\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[24 - (-5)] \text{°C}}{0.06155 \text{ °C/W}} = 471 \text{ W}
\]

The inner surface temperature of the window glass can be determined from
\[
\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}} \quad \rightarrow \quad T_1 = T_{\infty 1} - \dot{Q}R_{\text{conv},1} = 24 \text{°C} - (471 \text{ W})(0.04167 \text{ °C/W}) = 4.4 \text{°C}
\]
Heat is to be conducted along a circuit board with a copper layer on one side. The percentages of heat conduction along the copper and epoxy layers as well as the effective thermal conductivity of the board are to be determined.

**Assumptions**
1. Steady operating conditions exist.
2. Heat transfer is one-dimensional since heat transfer from the side surfaces is disregarded.
3. Thermal conductivities are constant.

**Properties**
The thermal conductivities are given to be $k = 386 \text{ W/m} \cdot \text{°C}$ for copper and $0.26 \text{ W/m} \cdot \text{°C}$ for epoxy layers.

**Analysis**
We take the length in the direction of heat transfer to be $L$ and the width of the board to be $w$. Then heat conduction along this two-layer board can be expressed as

$$Q = Q_{\text{copper}} + Q_{\text{epoxy}} = \left( k_A \frac{\Delta T}{L} \right)_{\text{copper}} + \left( k_A \frac{\Delta T}{L} \right)_{\text{epoxy}}$$

$$= \left[ (kt)_{\text{copper}} + (kt)_{\text{epoxy}} \right] w \frac{\Delta T}{L}$$

Heat conduction along an “equivalent” board of thickness $t = t_{\text{copper}} + t_{\text{epoxy}}$ and thermal conductivity $k_{\text{eff}}$ can be expressed as

$$Q = \left( k_A \frac{\Delta T}{L} \right)_{\text{board}} = k_{\text{eff}} (t_{\text{copper}} + t_{\text{epoxy}}) w \frac{\Delta T}{L}$$

Setting the two relations above equal to each other and solving for the effective conductivity gives

$$k_{\text{eff}} (t_{\text{copper}} + t_{\text{epoxy}}) = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} \quad \Rightarrow \quad k_{\text{eff}} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}}$$

Note that heat conduction is proportional to $kt$. Substituting, the fractions of heat conducted along the copper and epoxy layers as well as the effective thermal conductivity of the board are determined to be

$$(kt)_{\text{copper}} = (386 \text{ W/m} \cdot \text{°C})(0.0001 \text{ m}) = 0.0386 \text{ W/°C}$$

$$(kt)_{\text{epoxy}} = (0.26 \text{ W/m} \cdot \text{°C})(0.0012 \text{ m}) = 0.000312 \text{ W/°C}$$

$$(kt)_{\text{total}} = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} = 0.0386 + 0.000312 = 0.038912 \text{ W/°C}$$

$$f_{\text{epoxy}} = \frac{(kt)_{\text{epoxy}}}{(kt)_{\text{total}}} = \frac{0.000312}{0.038912} = 0.08 = 0.8\%$$

$$f_{\text{copper}} = \frac{(kt)_{\text{copper}}}{(kt)_{\text{total}}} = \frac{0.0386}{0.038912} = 0.992 = 99.2\%$$

and

$$k_{\text{eff}} = \frac{(386 \times 0.0001 + 0.26 \times 0.0012) \text{ W/°C}}{(0.0001 + 0.0012) \text{ m}} = 29.9 \text{ W/m} \cdot \text{°C}$$
**Chapter 3, Solution 58E.**

A wall is to be constructed using solid bricks or identical size bricks with 9 square air holes. There is a 0.5 in thick sheetrock layer between two adjacent bricks on all four sides, and on both sides of the wall. The rates of heat transfer through the wall constructed of solid bricks and of bricks with air holes are to be determined.

**Assumptions**

1. Heat transfer is steady since there is no indication of change with time.
2. Heat transfer through the wall is one-dimensional.
3. Thermal conductivities are constant.
4. Heat transfer coefficients account for the radiation heat transfer.

**Properties**

The thermal conductivities are given to be \( k = 0.40 \text{ Btu/h-ft-}^\circ\text{F} \) for bricks, \( k = 0.015 \text{ Btu/h-ft-}^\circ\text{F} \) for air, and \( k = 0.10 \text{ Btu/h-ft-}^\circ\text{F} \) for sheetrock.

**Analysis**

(a) The representative surface area is \( A = (7.5/12)(7.5/12) = 0.3906 \text{ ft}^2 \). The thermal resistance network and the individual thermal resistances if the wall is constructed of solid bricks are

\[
\begin{align*}
R_1 &= \frac{1}{h_i A} = \frac{1}{(1.5 \text{ Btu/h-ft}^2 \cdot ^\circ\text{F})(0.3906 \text{ ft}^2)} = 1.7068 \text{ h}^\circ\text{F/Btu} \\
R_3 &= \frac{L}{kA} = \frac{0.5/12 \text{ ft}}{(0.10 \text{ Btu/h-ft} \cdot ^\circ\text{F})(0.3906 \text{ ft}^2)} = 1.0667 \text{ h} \cdot ^\circ\text{F/Btu} \\
R_2 &= \frac{L}{kA} = \frac{9/12 \text{ ft}}{(0.10 \text{ Btu/h-ft} \cdot ^\circ\text{F})(7/12)(0.5/12)\text{ ft}^2} = 288 \text{ h} \cdot ^\circ\text{F/Btu} \\
R_4 &= \frac{L}{kA} = \frac{9/12 \text{ ft}}{(0.40 \text{ Btu/h-ft} \cdot ^\circ\text{F})(7/12)(7/12)\text{ ft}^2} = 5.51 \text{ h} \cdot ^\circ\text{F/Btu} \\
R_o &= \frac{1}{h_o A} = \frac{1}{(4 \text{ Btu/h-ft}^2 \cdot ^\circ\text{F})(0.3906 \text{ ft}^2)} = 0.64 \text{ h} \cdot ^\circ\text{F/Btu} \\
\frac{1}{R_{mid}} &= \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{288} + \frac{1}{308.57} + \frac{1}{5.51} \Rightarrow R_{mid} = 5.3135 \text{ h} \cdot ^\circ\text{F/Btu} \\
R_{total} &= R_1 + R_3 + R_{mid} + R_4 = 1.7068 + 1.0667 + 5.3135 + 1.0667 + 0.64 = 9.7937 \text{ h} \cdot ^\circ\text{F/Btu} \\
\dot{Q} &= \frac{T_{w1} - T_{w2}}{R_{total}} = \frac{(80-30) ^\circ\text{F}}{9.7937 \text{ h} \cdot ^\circ\text{F/Btu}} = 5.1053 \text{ Btu/h}
\end{align*}
\]

Then steady rate of heat transfer through entire wall becomes

\[
\dot{Q}_{total} = (5.1053 \text{ Btu/h}) \cdot \frac{(30 \text{ ft})(10 \text{ ft})}{0.3906 \text{ m}^2} = 3921 \text{ Btu/h}
\]

(b) The thermal resistance network and the individual thermal resistances if the wall is constructed of bricks with air holes are
Then steady rate of heat transfer through entire wall becomes

\[
\dot{Q}_{total} = \frac{(3.817 \text{ Btu/h}) (30 \text{ ft})(10 \text{ ft})}{0.3906 \text{ ft}^2} = 2932 \text{ Btu/h}
\]
Chapter 3, Solution 74.

A 50-m long section of a steam pipe passes through an open space at 15°C. The rate of heat loss from the steam pipe, the annual cost of this heat loss, and the thickness of fiberglass insulation needed to save 90 percent of the heat lost are to be determined.

**Assumptions** 1 Heat transfer is steady since there is no indication of any change with time. 2 Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. 3 Thermal conductivity is constant. 4 The thermal contact resistance at the interface is negligible. 5 The pipe temperature remains constant at about 150°C with or without insulation. 6 The combined heat transfer coefficient on the outer surface remains constant even after the pipe is insulated.

**Properties** The thermal conductivity of fiberglass insulation is given to be \( k = 0.035 \text{ W/m} \cdot \text{°C} \).

**Analysis**

(a) The rate of heat loss from the steam pipe is

\[
A_o = \pi DL = \pi (0.1 \text{ m})(50 \text{ m}) = 15.71 \text{ m}^2
\]

\[
\dot{Q}_{\text{bare}} = h_o (T_s - T_{\text{air}}) = (20 \text{ W/m}^2 \cdot \text{°C})(15.71 \text{ m}^2)(150 - 15)\text{°C} = 42,412 \text{ W}
\]

(b) The amount of heat loss per year is

\[
\dot{Q} = \dot{Q}_{\Delta t} = (42,412 \text{ kJ/s})(365 \times 24 \times 3600 \text{ s/yr}) = 1.337 \times 10^9 \text{ kJ/yr}
\]

The amount of gas consumption from the natural gas furnace that has an efficiency of 75% is

\[
Q_{\text{gas}} = \frac{1.337 \times 10^9 \text{ kJ/yr}}{0.75} \left( \frac{1 \text{ therm}}{105,500 \text{ kJ}} \right) = 16,903 \text{ therms/yr}
\]

The annual cost of this energy lost is

Energy cost = (Energy used)(Unit cost of energy)

= (16,903 therms/yr)($0.52 / therm) = $8790/yr

(c) In order to save 90% of the heat loss and thus to reduce it to 0.1\times42,412 = 4241 \text{ W}, the thickness of insulation needed is determined from

\[
\dot{Q}_{\text{insulated}} = \frac{T_s - T_{\text{air}}}{R_o + R_{\text{insulation}}} = \frac{T_s - T_{\text{air}}}{h_o A_o} + \frac{1}{2\pi k L} \ln\left( \frac{r_2}{r_1} \right)
\]

Substituting and solving for \( r_2 \), we get

\[
4241 \text{ W} = \frac{(150 - 15)\text{°C}}{(20 \text{ W/m}^2 \cdot \text{°C})(2\pi(50 \text{ m})]} + \frac{\ln(r_2 / 0.05)}{2\pi(0.035 \text{ W/m} \cdot \text{°C})(50 \text{ m})}
\]

\[\longrightarrow r_2 = 0.0692 \text{ m} \]

Then the thickness of insulation becomes

\[
t_{\text{insulation}} = r_2 - r_1 = 6.92 - 5 = 1.92 \text{ cm}
\]
Chapter 3, Solution 116.

A commercially available heat sink is to be selected to keep the case temperature of a transistor below 55°C in an environment at 18°C.

Assumptions 1 Steady operating conditions exist. 2 The transistor case is isothermal at 55°C. 3 The contact resistance between the transistor and the heat sink is negligible.

Analysis The thermal resistance between the transistor attached to the sink and the ambient air is determined to be

\[
\dot{Q} = \frac{\Delta T}{R_{\text{case-ambient}}} = \frac{T_{\text{transistor}} - T_{\infty}}{\dot{Q}} = \frac{(55 - 18)\degree C}{25\text{ W}} = 1.5\degree C/\text{W}
\]

The thermal resistance of the heat sink must be below 1.5°C/W. Table 3-6 reveals that HS5030 in both horizontal and vertical positions, HS6071 in vertical position, and HS6115 in both horizontal and vertical positions can be selected.
Chapter 3, Solution 121.

A circuit board houses 80 logic chips on one side, dissipating 0.04 W each through the back side of the board to the surrounding medium. The temperatures on the two sides of the circuit board are to be determined for the cases of no fins and 864 aluminum pin fins on the back surface.

**Assumptions** 1 Steady operating conditions exist. 2 The temperature in the board and along the fins varies in one direction only (normal to the board). 3 All the heat generated in the chips is conducted across the circuit board, and is dissipated from the back side of the board. 4 Heat transfer from the fin tips is negligible. 5 The heat transfer coefficient is constant and uniform over the entire fin surface. 6 The thermal properties of the fins are constant. 7 The heat transfer coefficient accounts for the effect of radiation from the fins.

**Properties** The thermal conductivities are given to be $k = 30 \text{ W/m} \cdot \text{°C}$ for the circuit board, $k = 237 \text{ W/m} \cdot \text{°C}$ for the aluminum plate and fins, and $k = 1.8 \text{ W/m} \cdot \text{°C}$ for the epoxy adhesive.

**Analysis**  
(a) The total rate of heat transfer dissipated by the chips is  
\[ Q = 80 \times (0.04 \text{ W}) = 3.2 \text{ W} \]

The individual resistances are  
\[ R_{\text{board}} = \frac{L}{kA} = \frac{0.003 \text{ m}}{(30 \text{ W/m} \cdot \text{°C})(0.0216 \text{ m}^2)} = 0.00463 \text{ °C/W} \]
\[ R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(40 \text{ W/m}^2 \cdot \text{°C})(0.0216 \text{ m}^2)} = 1.1574 \text{ °C/W} \]
\[ R_{\text{total}} = R_{\text{board}} + R_{\text{conv}} = 0.00463 + 1.1574 = 1.1620 \text{ °C/W} \]

The temperatures on the two sides of the circuit board are  
\[ Q = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} = T_{\infty 2} + \dot{Q}R_{\text{total}} = 40\degree \text{C} + (3.2 \text{ W})(1.1620 \text{ °C/W}) = 43.7\degree \text{C} \]
\[ \dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} = T_1 - \dot{Q}R_{\text{board}} = 43.7\degree \text{C} - (3.2 \text{ W})(0.00463 \text{ °C/W}) = 43.7 - 0.015 = 43.7\degree \text{C} \]

Therefore, the board is nearly isothermal.

(b) Noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be  
\[ m = \sqrt{\frac{hp}{kA_f}} = \sqrt{\frac{h \pi D}{k \pi D^2 / 4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(40 \text{ W/m}^2 \cdot \text{°C})}{(237 \text{ W/m} \cdot \text{°C})(0.0025 \text{ m})}} = 16.43 \text{ m}^{-1} \]
\[ \eta_{\text{fin}} = \frac{\tan h mL}{mL} = \tan h(16.43 \text{ m}^{-1} \times 0.02 \text{ m}) = 0.965 \]

The fins can be assumed to be at base temperature provided that the fin area is modified by multiplying it by 0.965. Then the various thermal resistances are
\[ R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.0002 \text{ m}}{(1.8 \text{ W/m}.{ }^\circ\text{C})(0.0216 \text{ m}^2)} = 0.0051 ^\circ\text{C/W} \]

\[ R_{\text{Al}} = \frac{L}{kA} = \frac{0.002 \text{ m}}{(237 \text{ W/m}.{ }^\circ\text{C})(0.0216 \text{ m}^2)} = 0.00039 ^\circ\text{C/W} \]

\[ A_{\text{finned}} = \eta_{\text{fin}}n\pi Dl = 0.965 \times 864 \pi (0.0025 \text{ m})(0.02 \text{ m}) = 0.131 \text{ m}^2 \]

\[ A_{\text{unfinned}} = 0.0216 - 864 \frac{\pi D^2}{4} = 0.0216 - 864 \times \frac{\pi (0.0025)^2}{4} = 0.0174 \text{ m}^2 \]

\[ A_{\text{total, with fins}} = A_{\text{finned}} + A_{\text{unfinned}} = 0.131 + 0.0174 = 0.148 \text{ m}^2 \]

\[ R_{\text{conv}} = \frac{1}{hA_{\text{total, with fins}}} = \frac{1}{(40 \text{ W/m}^2 \cdot ^\circ\text{C})(0.148 \text{ m}^2)} = 0.1689 ^\circ\text{C/W} \]

\[ R_{\text{total}} = R_{\text{board}} + R_{\text{epoxy}} + R_{\text{aluminum}} + R_{\text{conv}} \]

\[ = 0.00463 + 0.0051 + 0.00039 + 0.1689 = 0.1790 ^\circ\text{C/W} \]

Then the temperatures on the two sides of the circuit board becomes

\[ \dot{Q} = \frac{T_1 - T_{\text{in}}}{R_{\text{total}}} \rightarrow T_1 = T_{\text{in}} + \dot{Q}R_{\text{total}} = 40 ^\circ\text{C} + (3.2 \text{ W})(0.1790 ^\circ\text{C/W}) = 40.6 ^\circ\text{C} \]

\[ \dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \rightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 40.6 ^\circ\text{C} - (3.2 \text{ W})(0.00463 ^\circ\text{C/W}) = 40.6 - 0.015 \equiv 40.6 ^\circ\text{C} \]